’t Hooft Anomaly,
Symmetry breaking, Gaplessness

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[arXiv:1907.xxxxx   KO, Clay Córdova]

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’t Hooft anomaly matching

- ’t Hooft anomaly: quantum modification of global symmetry transformation
- **’t Hooft anomaly matching** : anomaly(UV) = anomaly(IR)
- Often computable in UV, even if IR physics is unknown
- Constrains possible IR physics
- e.g.: pure $\mathrm{SU}(2)$ Yang-Mills at $\theta = \pi$ **[Gaiotto Kapustin Komargodski Seiberg ’17]**
  This talk: **New anomaly constraint**

't Hooft anomaly and IR physics

Q: Given an anomaly, what is the possible IR behavior?

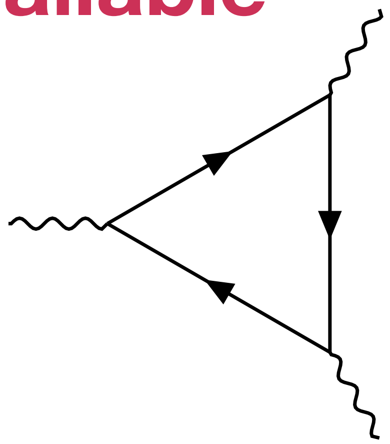
- Nontrivial anomaly ➡ ✖ trivial IR fixed pt
- $\begin{cases} \text{Gapless} \Rightarrow \text{CFT} \\ \text{Topological d.o.f.} \Rightarrow \text{Non-trivial TQFT :} \end{cases}$

  Not always available

- e.g.: Local (triangle) anomaly

  $\langle \partial jjj \rangle = (\partial \delta)^2$ ➡ $\langle j(x)j(y)j(z) \rangle \neq 0$

  ➡ gapless IR (CFT)



't Hooft anomaly and IR physics (2)

- Discrete symmetry → { Gapless (CFT) / Spontaneous Breaking / Symmetry presering TQFT }
  Exists?
- 2 dimensions → gapless or SSB **[Cheng, Gu, Wen '10]**
  No interesting TQFT with single vacuum
- d = 3 T $\times (-1)^F$ mixed anomaly: $\mathbb{Z}_{16}$ (top. superconductor)
  N majrana ψ: N mod 16
  [Metlitski + Fidkowski, Chen, Vishwanath '13-'14], [Wang, Senthil '14],[Kapustin, Thorngren, Turzillo '14], [Kitaev '15], [Hsieh, Cho, Ryu '15] …
  ∃ T-preserving TQFT matching anomaly: $SO(N)_N \leftrightarrow SO(N)_{-N}$
  [Aharony, Benini, Hsin, Seiberg '16], [Cheng '17], [Gomis, Komargodski, Seiberg '17], [Cordova, Hsin, Seiberg '17]…

’t Hooft anomaly and IR physics (2)

- d>2 any anomaly for unitary finite (ordinary, 0-form) symmetry in bosnic system

  ➡ a symmetry preserving TQFT [Witten ’16], [Wang,Wen,Witten ’17]

  Generalizes to a certain class of fermonic anomalies

  [Cheng ’18], [Fidkowski, Vishwanath, Metlitski ’18], [Guo, KO, Putrov, Wan, Wang ’19] …
  [Kobayashi, KO, Tachikawa ’19], [Kobayashi ’19]

- Most anomalies for (ordinary, 0-form) symmetry in d>2 **admits symmetry preserving TQFT.**

Main result

Remaining situation:

- global (discrete) anomaly for continuous symmetry
- discrete center (generalized, 1-form) symmetry

[Gaiotto, Kapustin, Seiberg, Willett '14]

Q: Can we find a **clear sufficient condition** for a 't Hooft anomaly to rule out the possibility of sym. pres. TQFT?

Main result

Remaining situation:

$\begin{cases} \text{global (discrete) anomaly for continuous symmetry} \\ \text{discrete center (generalized, 1-form) symmetry} \end{cases}$

[Gaiotto, Kapustin, Seiberg, Willett ’14]

Q: Can we find a **clear sufficient condition** for a ’t Hooft anomaly to rule out the possibility of sym. pres. TQFT?

YES!

The condition is rather complicated.

→ concrete consequences

Concrete examples (continuous)

- Witten Anomaly for $\pi_d(G)$
  - $\pi_4(\mathrm{SU}(2)) = \mathbb{Z}_2$
  - A single Weyl $\psi_{\mathbf{fund}}$ in 4d
  - No sym. pres. TQFT [García-Etxebarria, Hayashi, KO, Tachikawa, Yonekura '17]
  - = No gapped phase (in particular, no mass term)

Concrete examples (2) (continuous)

- 3d Parity anomaly: $G \times \mathrm{T}$
  - Majorana ψ → $\frac{1}{2}\mathrm{CS}$ counter term to preserve T
  - No sym. pres. TQFT if ∃instanton on S^4 w/ instanton # =1
  - $\mathrm{SU}(2)$: no TQFT w/ single vacuum → CFT or 2 vacua
    $\mathrm{SO}(3)$: $\mathrm{U}(1)_2$ CS

Concrete example (discrete)

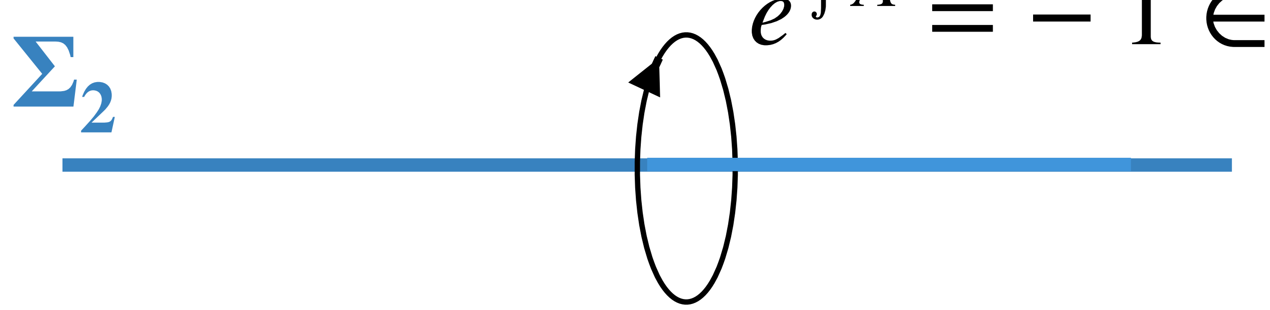
- $\mathrm{SU}(2)$ Yang-Mills at $\theta = \pi$
  - $\mathrm{T} \times \mathbb{Z}_2^{\text{center}}$ symmetry with mixed anomaly

    [Gaiotto Kapustin Komargodski Seiberg '17]
  - **No sym. pres. TQFT**
  - No confined gapped phase with single vacuum
  - Possible scenarios $\begin{cases} \text{gapless} \\ \text{T-breaking 2 vacua (confined)} \\ \text{deconfined gapped phase} \end{cases}$

Center symmetry of $SU(2)$ YM

- $Z(\mathrm{SU}(2)) = \mathbb{Z}_2$
- Wilson line (**order param. of confinement**) in **fund** is charged
- Confinement = preservation of $\mathbb{Z}_2^{\text{center}}$
- **Charge op.** $(-1)^{Q[\Sigma_2]}$ **:** topological, codim. 2
  Holonomy in center along a circle encircling Σ_2
  ('t Hooft twisted boundary condition)

$$e^{\int A} = -1 \in \mathrm{SU}(2)$$



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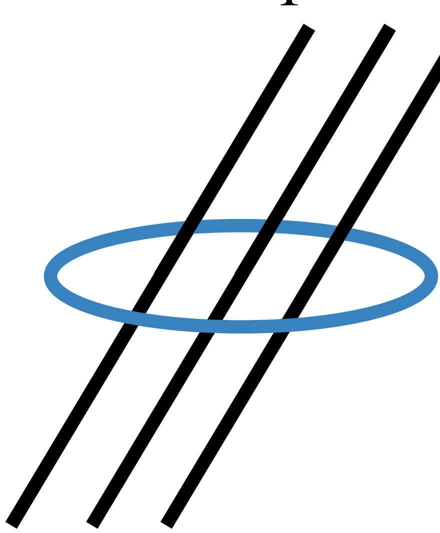
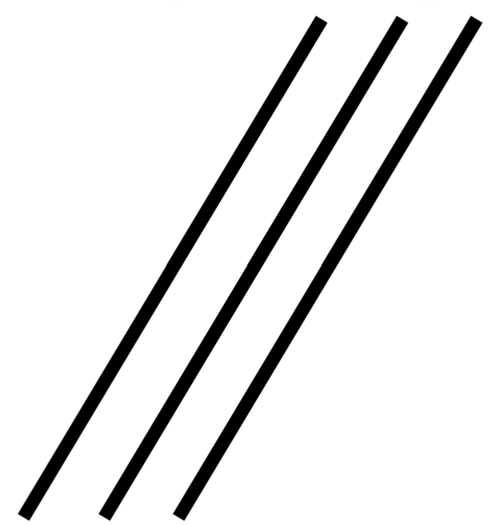
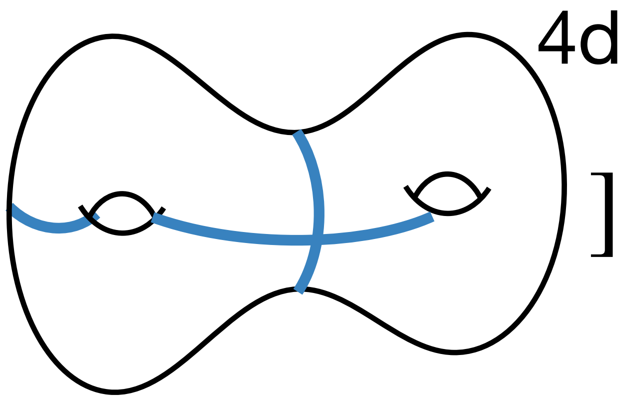
$W[L_1], W[L_2], \cdots$

$(-1)^{Q[\Sigma_2]}$  $=$ $(-1)^{\#\text{ of } W}$ 

$W[L_1], W[L_2], \cdots$

Sym. twisted partition function

- $Z[$  $] = \int_{\text{hol. around op.}} \mathscr{D}A \;\; e^{-S}$

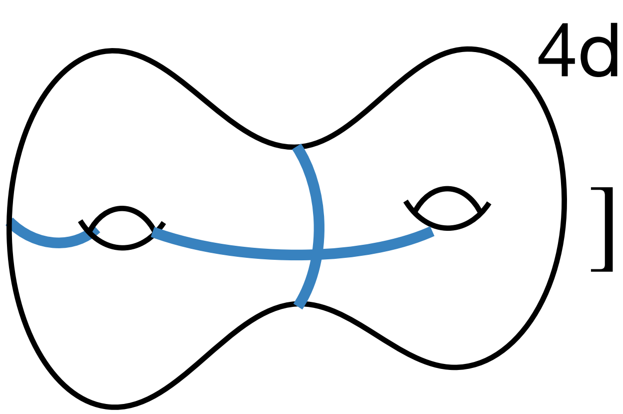
  ('t Hooft twisted boundary condition)

- #(Instanton): $\frac{1}{8\pi^2}\int_{M^4_{\text{spin}}} \mathrm{tr} F \wedge F \in \mathbb{Z} \;+ \frac{1}{2}$ #(**intersections**)

  Fractional instanton

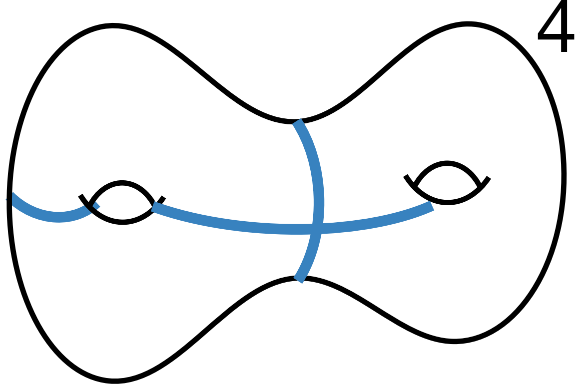
- Theta term at $\theta = \pi : e^{-S} = e^{\pi i \#(\text{instanton})} \times (\text{real})$

  $= (-1)^{\frac{1}{2}\#(\textbf{intersections})} \times (\text{real})$

- $Z[$  $] \in \begin{cases} \mathbb{R} & \#(\textbf{intersections}) : \text{even} \\ i\mathbb{R} & \#(\textbf{intersections}) : \text{odd} \end{cases}$

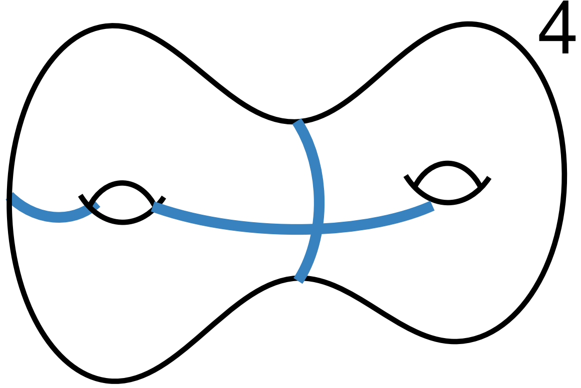
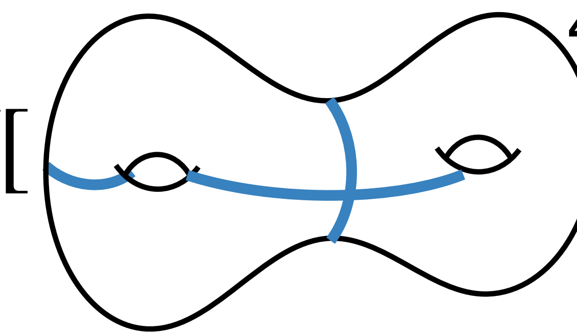
$\mathsf{T} \times \mathbb{Z}_2^{\text{center}}$ anomaly

[Gaiotto Kapustin Komargodski Seiberg '17]

- $Z[$  $]$ $\in \begin{cases} \mathbb{R} & \#(\text{intersections}) : \text{even} \\ i\mathbb{R} & \#(\text{intersections}) : \text{odd} \end{cases}$

  4d

  Anomaly !

- $Z[\mathsf{T}$  $] = (-1)^{\#(\text{intersections})}\, Z[$  $]$

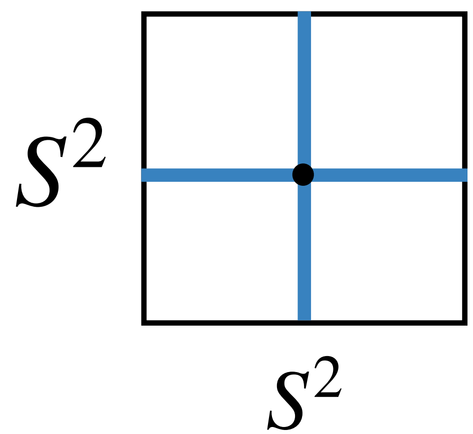
  4d   4d

> Q: Can this behavior be reproduced by a symmetry-preserving TQFT?

Specializing the manifold

- Take the manifold to be $M^4 = S^2 \times S^2$, and
- put two charge operators each wrapping each S^2

$$M^4_{\text{twist}} = (S^2 \times S^2)_{\text{twist}}$$



$$Z[\mathrm{T}(M^4_{\text{twist}})] = (-1)^{\#(\text{intersections})}\, Z[M^4_{\text{twist}}]$$

- #(**intersections**)=1, $\mathrm{T}(M^4_{\text{twist}}) = M^4_{\text{twist}}$

$$Z[M^4_{\text{twist}}] = 0$$

Compatible with sym. pres. TQFT?

Charge op. in sym. pres. TQFT

- The charge operator $(-1)^{Q[\Sigma_2]}$ must be nontrivial in IR theory

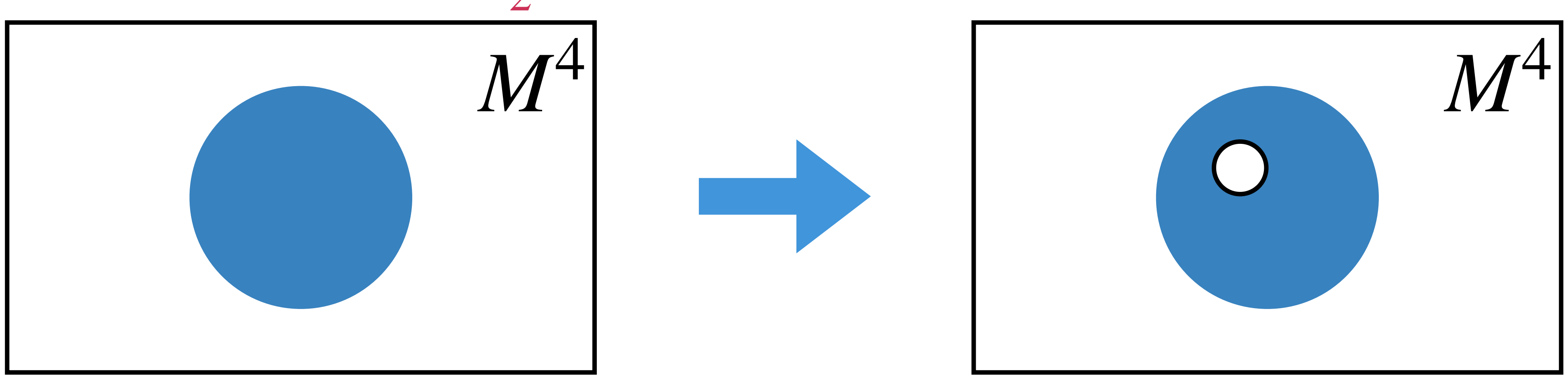
- center sym. pres. = confinement

  ➡ Wilson line: area law = absent in IR if **TQFT**

- $(-1)^{Q[\Sigma_2]}$ is transparent to line op.s in IR

- ∃boundary condition with which $(-1)^{Q[\Sigma_2]} = (-1)^{Q[\Sigma_2 - D^2]}$

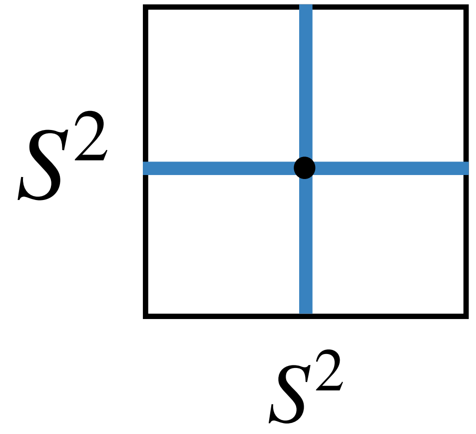
if $\mathbb{Z}_2^{\text{center}}$ preserved and TQFT



No symmetry-preserving TQFT

$$Z[M^4_{\text{twist}}] \stackrel{?}{=} 0$$

Compatible with sym. pres. TQFT?



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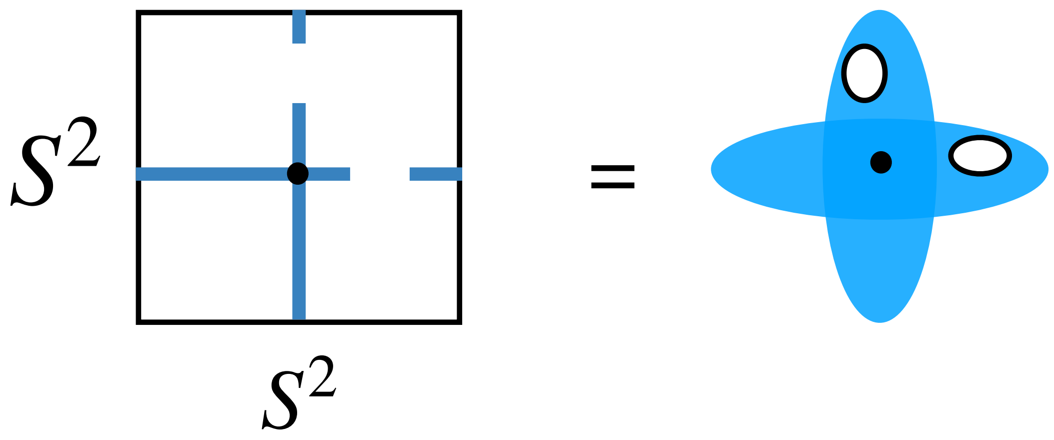

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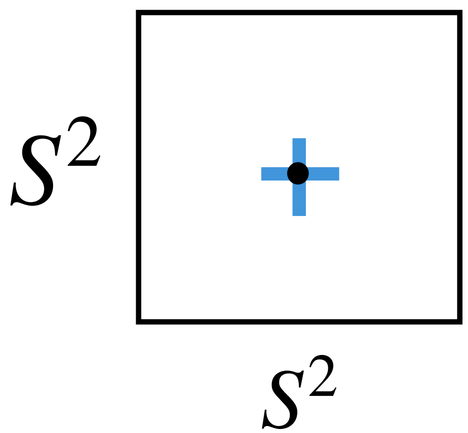
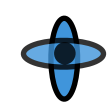


$=$ 

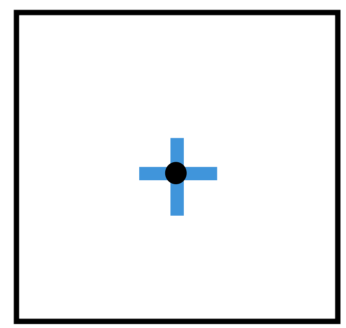
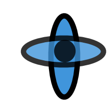
$= \langle X \rangle_{S^2 \times S^2}$ X: local op.

Single vacuum ⟹ $X = \text{(phase)} \times \mathbb{1}$

$$|\,Z[M^4_{\text{twist}}]\,| = |\,Z[S^2 \times S^2]\,| > 0$$ from Unitarity

No symmetry-preserving TQFT

$$Z[M^4_{\text{twist}}] \overset{?}{=} 0$$
Compatible with sym. pres. TQFT?

S^2  S^2 $=$ 

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Single vacuum ➡ $X = (\text{phase}) \times \mathbb{1}$

$|Z[M^4_{\text{twist}}]| = |Z[S^2 \times S^2]| > 0$ from Unitarity

$Z[M^4_{\text{twist}}] \neq 0$ **in sym. pres. TQFT!**
No confined gapped phase with single vacuum

Recap.

- $\mathrm{SU}(2)$ Yang-Mills at $\theta = \pi$
  - $\mathsf{T} \times \mathbb{Z}_2^{\text{center}}$ symmetry with mixed anomaly

    [Gaiotto Kapustin Komargodski Seiberg '17]
  - No sym. pres. TQFT
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Continuous symmetry

- TQFT has a discrete extended operator spectrum
  $\Rightarrow$ (Generalized) symmetry in TQFT is discrete
- No nontrivial (generalized) symmetry twist on S^d.
- Cannot mimic a nontrivial continuous sym. b.g. on S^d
- If $\exists$anomalous transformation law of $Z[S^d, A]$,
  it cannot be matched by a symmetry preserving TQFT.
- Witten anomaly, Parity anomaly

Summary

- ’t Hooft anomaly sometimes forces SSB or gapless phase

Q: When?

- $\mathrm{SU}(2)$ Yang-Mills at $\theta = \pi$
  - No symmetry preserving TQFT matching the anomaly
- Generalizable using the language of generalized sym.
  [Gaiotto, Kapustin, Seiberg, Willett ’14]
  - Anomalous transformation law on $S^{k_1} \times S^{k_2}$
  - $\mathrm{SU}(2)$ adj QCD with 2ψ: $\mathbb{Z}_8^{\text{axial}} \times \mathbb{Z}_2^{\text{center}}$ anomaly
    [Anber, Poppitz ’18], [Córdova, Dumitrescu ’18], [Bi, Senthil ’18], [Wan, Wang ’18]
- Continuous symmetry anomaly is hard to match by symmetry preserving TQFT.

Outlook

- **Far from perfect ! Necessary and sufficient condition?**
  - $Z_k \times$ gravity^2 anomaly of fermion in 4d?
  - $Z[K3] = 0$
- Given any two theory with same 't Hooft anomaly,
  ∃ Up & down RG flow?
  (SUSY: no, Witten index.)

Thank you !