A toy model for the Kerr/CFT correspondence

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Motivation

- *universal entropy for black holes*

$$S_{BH} = \frac{\mathcal{A}_H}{4G\hbar}$$

AdS_3/CFT_2

- *good microscopic understanding only for black holes with AdS_3 factor in the near-horizon (charged, supersymmetric)*

    - *infinite-dimensional conformal symmetry (2 copies of Virasoro algebra)*

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{24}m(m^2-1)\delta_{m+n} \qquad m, n \in \mathbb{Z}$$

    - *universal entropy formula*

$$S_{Cardy} = \frac{\pi^2}{3}c\,(T_L + T_R)$$

- *realistic black holes: Kerr → mass M and angular momentum J*

- *most progress for extremal Kerr $M^2 = J$: Kerr/CFT correspondence*

GRS 105+1915, black hole in Cygnus X-1

(Virasoro symmetry)

Plan

- *review of the Kerr/CFT correspondence*
- *puzzles → no dynamics*

  → second copy of Virasoro
- *string-theoretical toy model I: both puzzles solved!*

  → Virasoro x Virasoro acts on entire linearized phase space
- *string-theoretical toy model II: "travelling waves"*

  → background unstable
- *conclusions*

The Kerr/CFT correspondence

MG, Hartman, Song, Strominger '08

- near-horizon geometry of the extreme Kerr black hole (NHEK)

$$\Omega^2(\theta) = \frac{1+\cos^2\theta}{2}$$

$$ds^2 = 2J\,\Omega^2(\theta)\Big[\underbrace{-r^2dt^2 + \frac{dr^2}{r^2}}_{AdS_2} + \frac{\sin^2\theta}{\Omega^4(\theta)}\underbrace{(d\phi + r dt)^2}_{U(1)\ \text{fibre}} + d\theta^2\Big]$$

Bardeen, Horowitz '99

AdS_2 ← $U(1)$ fibre

$\phi \sim \phi + 2\pi$

- self-dual spacelike warped AdS_3 θ - dependent: stretched/ squashed
- isometry $SL(2,\mathbb{R})_L \times U(1)_R$ → Virasoro!

$$\xi_n = e^{in\phi}(\partial_\phi + inr\partial_r)$$

$$c = 12J$$

- Cardy entropy → "chiral half" of a CFT_2
- generalizes to all extremal black holes → universality!
- expect 2nd Virasoro that simultaneously enhances $SL(2,\mathbb{R})_L$ → elusive!

The “no dynamics” puzzle

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No dynamics in the extremal Kerr throat

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Abstract: Motivated by the Kerr/CFT conjecture, we explore solutions of vacuum general relativity whose asymptotic behavior agrees with that of the extremal Kerr throat, sometimes called the Near-Horizon Extreme Kerr (NHEK) geometry. We argue that all

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Kerr-CFT and gravitational perturbations

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Abstract: Motivated by the Kerr-CFT conjecture, we investigate perturbations of the near-horizon extreme Kerr spacetime. The Teukolsky equation for a massless field of arbitrary spin is solved. Solutions fall into two classes: normal modes and traveling waves.

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- *linearized perturbations in NHEK* $\varphi \sim r^{-h(\kappa)}\, e^{-i\omega t + i\kappa\phi}$
- *conformal dimensions* $h(\kappa)$ *: real → normal modes*

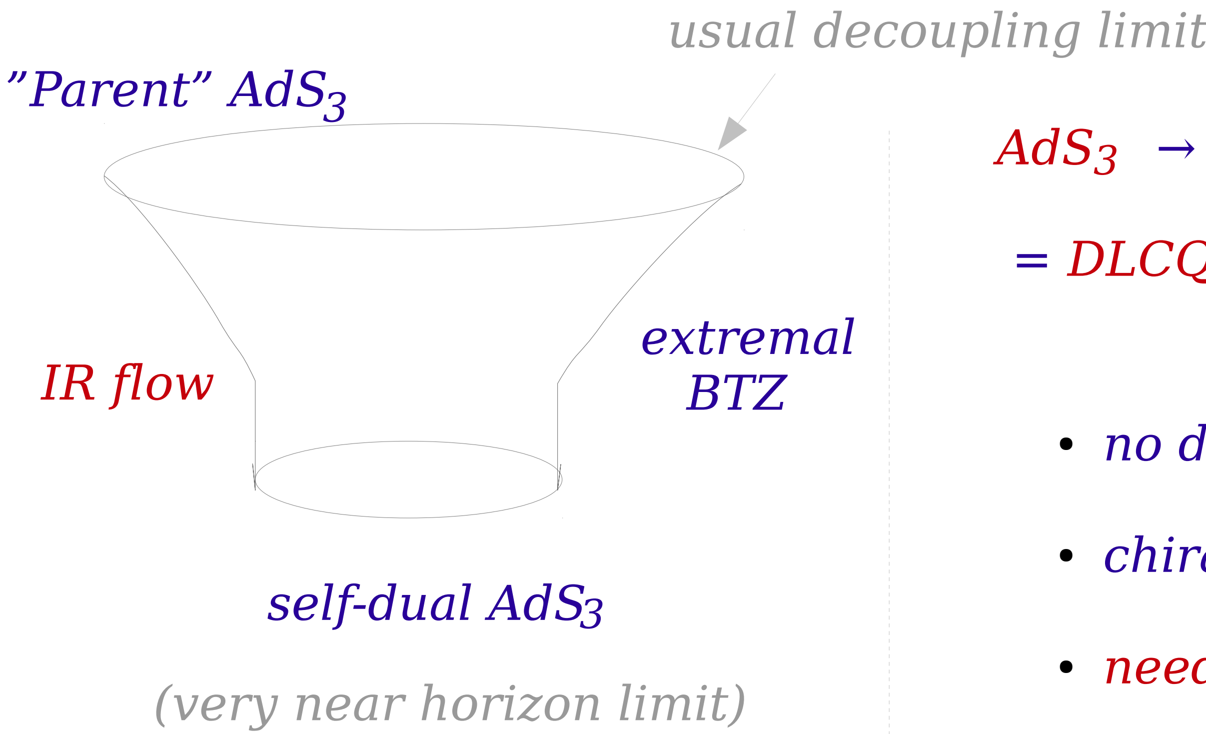
  - imaginary: “travelling waves” → superradiance!
- *backreaction destroys bnd. cond. on NHEK → finite energy in* AdS_2 *throat*

  → instability due to oscillatory modes
- *only boundary gravitons left → no dynamics!* *What does Cardy count?*

No dynamics and DLCQ

- *holographic understanding of "no dynamics" for self-dual AdS_3*

Balasubramanian, de Boer, Sheikh-Jabbari, Simon '09



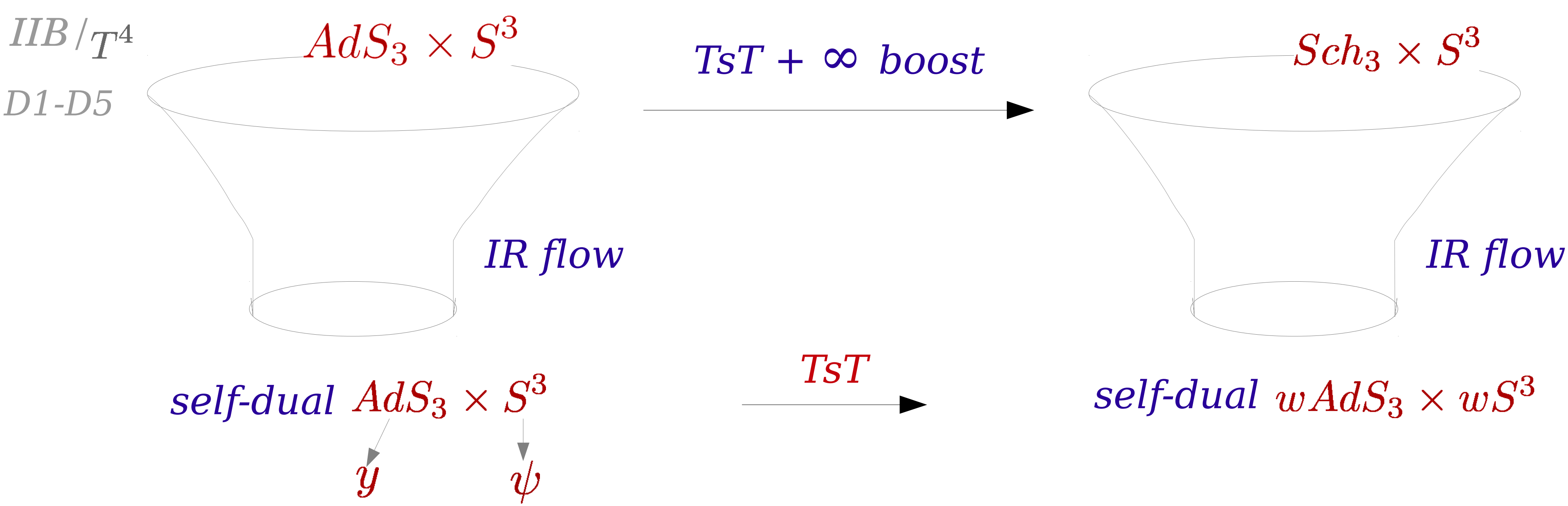
$AdS_3 \rightarrow$ self-dual AdS_3 flow

= DLCQ limit CFT_2: freezes left-movers

- *no dynamics*
- *chiral half of CFT_2*
- *need parent theory to derive Cardy*

- *"parent" space-time for NHEK?*
- *string theory embedding!*

String-theoretical construction of warped AdS_3



- TsT: T-duality along y , shift $\psi \to \psi + 2\lambda\,\tilde{y}$, T-duality back $\lambda \notin \mathbb{Z}$ B-field
- constant warping, entropy preserved (Cardy)
- other backgrounds with RR flux: $STsTS$, $T^4STsTST^4$ Bena, M.G, Song'12
    - near-horizon of extreme charged Myers-Perry $S = 2\pi\sqrt{J_L^2 - Q^3} = \pi^2\, c\, T_R/3$
    - S-dual dipole background M.G., Strominger'10
- Kerr/CFT correspondence = 3d Schrödinger holography (AdS/cold atom)

El-Showk, M.G '11

Toy model I

The S-dual dipole truncation

- consistent truncations type II B: $g_{\mu\nu},\ A_\mu,\ U$

$$S = \frac{1}{16\pi G_3}\int d^3x\sqrt{g}\left(R - 4(U)^2 - \frac{4}{\ell^2}\,e^{-4U}\,A^2 + \frac{2}{\ell^2}\,e^{-4U}(2 - e^{-4U}) - \frac{1}{\ell}\,\epsilon^{\mu\nu\rho}A_\mu F_{\nu\rho}\right)$$

Detournay, MG '12

- two propagating degrees of freedom: $A_\mu,\ U$

- vacuum solution: 3d Schrödinger space-time/ null warped AdS_3

$$ds^2 = \ell^2\left(-\lambda^2 r^2 du^2 + \frac{dr^2}{4r^2} + 2r\,du\,dv\right) \qquad A = \lambda\ell\, r\,du \qquad U = 0$$

- isometry $SL(2,\mathbb{R})_L \times U(1)_R$ → null

u: left-moving

v: right-moving

Plan: construct phase space ↔ space of solutions

- study its symmetries (two Virasoros?)

Finite-temperature solutions

Detournay, MG '12

- *warped BTZ black strings (T_L, T_R, λ) - very nice!*

$$\frac{ds^2_{wBTZ}}{\ell^2} = T_R^2\, dv^2 + 2r\, du\, dv + \left[T_L^2\,(1+\lambda^2 T_R^2) - \lambda^2 r^2\right] du^2 + \frac{(1+\lambda^2 T_R^2)\, dr^2}{4(r^2 - (T_L T_R)^2)}$$

$$A = \frac{\lambda \ell}{1+\lambda^2 T_R^2}\,(r du + T_R^2 dv) \qquad e^{4U} = 1 + \lambda^2 T_R^2$$

- *alternate writing:* $ds^2_{wBTZ} = (1+\lambda^2 T_R^2)\,\left(ds^2_{BTZ} - A \otimes A\right)$

- *thermodynamics/ unit length identical to BTZ black string* $x = u + v$

$$T_H,\ \Omega_H \qquad E \pm P = \frac{\pi}{6}\, c\, T^2_{L,R} \qquad c = c_{AdS}$$

- *Cardy formula for the entropy*

$$S_{wBTZ} = \frac{\pi}{6}\, c\,(T_L + T_R)$$

- *Limits* $T_R = 0$, $T_L = 0,\ i$ → *Poincaré/global null warped AdS*

Phase space

- bulk propagating modes → linearized perturbations (X modes)

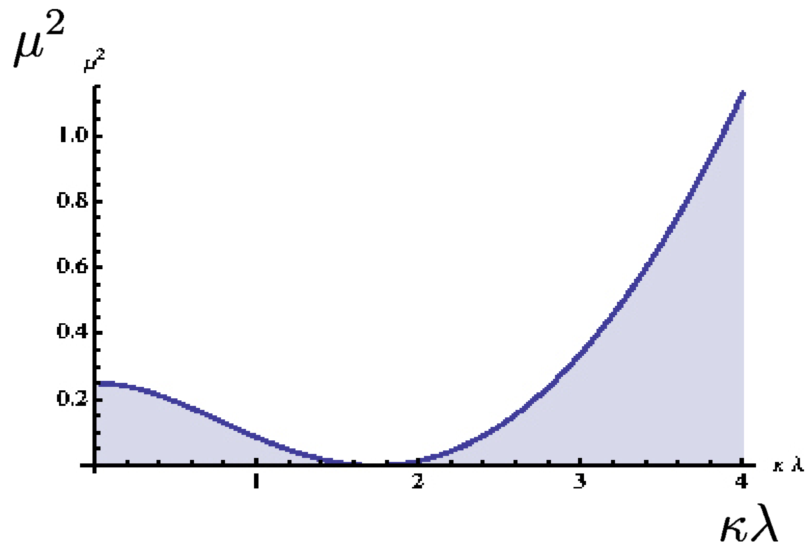
$$\varphi(u,v,r) \sim e^{-i\omega u + i\kappa v}\,\varphi(r)$$

$$U''(r) + \frac{2r}{r^2-(T_L T_R)^2}\,U'(r) + \frac{1}{r^2-(T_L T_R)^2}\left(\frac{1}{4} - \mu^2 + \frac{T_L^2\,\omega^2 + T_R^2\,\kappa^2 - 2r\omega\kappa}{4(r^2-(T_L T_R)^2)}\right) U(r) = 0$$

- all λ dependence in μ ; conformal dimension $h(\kappa) = \frac{1}{2} + \mu(\kappa)$
- two degrees of freedom → two possible values for μ

$$\mu = 1 \pm \frac{1}{2}\sqrt{1+\lambda^2\kappa^2}$$

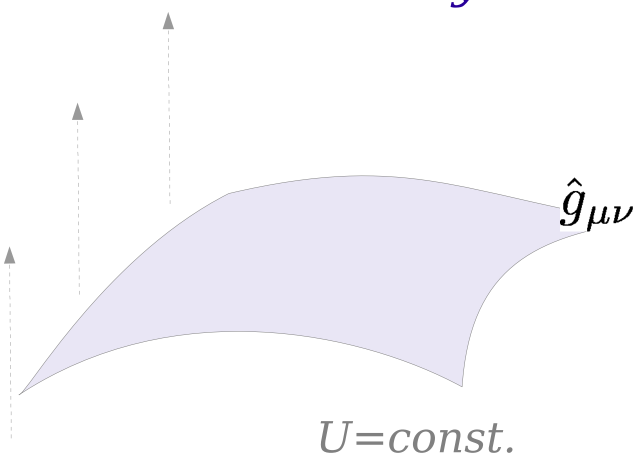
temperature-independent!



- boundary propagating modes : T-modes

The boundary propagating modes (T-modes)

- *locally diffeomorphic to the U=const solutions (black strings)*
- *characterized by U=const slice through phase space*



$$g_{\mu\nu} = e^{4U}\left(\hat{g}_{\mu\nu} - A_\mu A_\nu\right)$$

$$F = \tfrac{2}{\ell}\,\hat{\star} A \qquad \hat{A}^2 = e^{4U} - 1$$

→ *kills all propagating d.o.f*

$$\hat{\nabla}_\mu A_\nu + \hat{\nabla}_\nu A_\mu = 0$$

- $\hat{g}_{\mu\nu}$ *: AdS_3 metric* $\quad \hat{R}_{\mu\nu} + \frac{2}{\ell^2}\,\hat{g}_{\mu\nu} = 0$

  - boundary data in holographic renormalization — *M.G, '11, M.G. '13*

- *1-1 correspondence to solutions of 3d pure Einstein gravity*
- *non-local solution for* $A_\mu,\, g_{\mu\nu}$ *in terms of* $\hat{g}_{\mu\nu}$
- *full non-linear solution (explicit expression in skew gauge)*

Symplectic structure of T-mode phase space

- *phase space* $\leftrightarrow$ *space of solutions to the equations of motion*
- *presymplectic* $n-1$ *form* $\boldsymbol{\Theta}[\phi,\delta\phi]$

$$\delta\boldsymbol{\mathcal{L}}[\phi] = \boldsymbol{E}_\phi\delta\phi + \boldsymbol{d}\boldsymbol{\Theta}[\phi,\delta\phi]$$

- *symplectic form*

$$\boldsymbol{\omega}[\phi,\delta_1\phi,\delta_2\phi] = \delta_1\boldsymbol{\Theta}[\phi,\delta_2\phi] - \delta_2\boldsymbol{\Theta}[\phi,\delta_1\phi]$$

- *presymplectic form for S-dual dipole theory* $\boldsymbol{\Theta}_{\mu\nu} = \epsilon_{\mu\nu\lambda}\Theta^\lambda$

$$\Theta^\mu_{wAdS_3} = \underbrace{\nabla_\lambda h^{\lambda\mu} - \nabla^\mu h}_{\text{Einstein}} + \underbrace{\frac{2}{\ell}\epsilon^{\mu\nu\rho}A_\nu\delta A_\rho}_{\text{CS}} - \underbrace{8\nabla^\mu U\delta U}_{\text{scalar}}$$

- *ambiguity:* $\boldsymbol{\Theta} \to \boldsymbol{\Theta} + \boldsymbol{d}\boldsymbol{Y}[\phi,\delta\phi]$

$$\boldsymbol{\omega} \to \boldsymbol{\omega} + \boldsymbol{d}\left(\delta_1\boldsymbol{Y}[\phi,\delta_2\phi] - \delta_2\boldsymbol{Y}[\phi,\delta_1\phi]\right)$$

Equivalence of T-mode phase space to phase space of gravity in AdS_3

- *choose* $Y_\mu = -\epsilon_{\mu\alpha\beta} A^\alpha \delta A^\beta$
- *can show analytically that, on U=const slice*

$$\Theta_{wAdS_3} + dY = \hat{\epsilon}_{\mu\nu\rho}(\hat{\nabla}_\lambda \hat{h}^{\lambda\rho} - \hat{\nabla}^\rho \hat{h}) = \Theta_{AdS_3}$$

- *symplectic form on U=const slice:* $\omega_{wAdS_3} + d[\delta Y] = \omega_{AdS_3}$
- *conserved charges:* $\omega(\delta\phi, \mathcal{L}_\xi \phi) = d(\delta Q_\xi)$

Any consistent choice of boundary conditions in AdS_3

⇨ *consistent boundary conditions in warped AdS_3*

- *Brown-Henneaux (Dirichlet) boundary conditions*
- *mixed boundary conditions* Compere, Song, Strominger '13

1 ↔ 1 map between conserved charges in AdS_3 and in $wAdS_3$!

Including the propagating modes

- *conditions on symplectic form: normalizability and conservation*

$$\omega_{ur} \sim o(r^{-1}) \qquad \omega_{vr} \sim o(r^{-1}) \qquad \omega_{uv} \sim o(r^{0})$$

- *calculate:* $\omega_{tot} = \omega_{Einstein} + \omega_{CS} + \omega_{scalar} + \omega_{Y}$

- *contributions from: boundary gravitons* → $\mathcal{T}_L\,[F(u)]\,, \quad \mathcal{T}_R\,[G(v)]$

  - X-modes → $U(r) \sim r^{-\frac{1}{2}-\mu}\,, \qquad \mu \geq 0$

- *results:*

$$\omega_{ur}(\mathcal{T_L}, \mathcal{T_R}) \sim \omega_{vr}(\mathcal{T_L}, \mathcal{T_R}) \sim \mathcal{O}(r^{-3})$$

identical to AdS_3

$$\omega_{ur}(X_1, X_2) \sim \mathcal{O}(r^{-1-\mu_1-\mu_2})\,, \quad \omega_{uv}(X_1, X_2) \sim \mathcal{O}(r^{-\mu_1-\mu_2})$$

$$\omega_{ur}(\mathcal{T_R}, X) \sim \mathcal{O}(r^{-\frac{1}{2}-\mu})\,, \quad \omega_{uv}(\mathcal{T_R}, X) \sim \mathcal{O}(r^{\frac{1}{2}-\mu})$$

divergent!

$$0 \leq \mu \leq \frac{1}{2}$$

$$\omega_{ur}(\mathcal{T}_L, X) \sim \mathcal{O}(r^{-\frac{5}{2}-\mu})$$

Removing the divergences from the symplectic norm

- *found:* $\omega_{ur}(\mathcal{T}_{\mathcal{R}}, X)$, $\omega_{uv}(\mathcal{T}_{\mathcal{R}}, X)$ *divergent for* $\mu \in [0, 1/2]$

- *can cancel both divergences by boundary counterterm*

$$\omega \to \omega + d[\delta Y_{ct}]$$

$$Y^{ct}_{\mu}(\phi, \delta\phi) = A_{\mu}\, \delta f_1(U) + \epsilon_{\mu\nu\rho}\, \delta A^{\nu} \nabla^{\rho} f_2(U)$$

- Y_{ct} *does not contribute to* $\omega(\mathcal{T}, \mathcal{T})$

- *no finite contribution to* $\omega(\mathcal{T}, X)$, $\omega(X, X)$ → *positivity unaffected!*

- $f_{1,2}(U)$ *non-local functions of* κ → *compare with counterterms in holographic renormalization*

Partial conclusions

- *Virasoro x Virasoro symmetry can be made to act on entire gravity phase space!*
  - *non-linear level for T-modes*
  - *linear level for X-modes (around arbitrary T_R)*
- *non-linear effects unlikely to affect conclusion* $\mu \to 2\mu + 1/2$
- *if both Virasoros kept*

Mismatch to current understanding of field theory!!!

"dipole CFT" → non-local along v

→ only $SL(2,\mathbb{R})_L \times U(1)_R$ *invariance*

Toy model II - superradiance

The “NHEK” truncation

- 6d uplift of near-horizon of charged extreme 5d Myers-Perry ∈ II B/ T^4

$$S = 2\pi\sqrt{J_L^2 - Q^3}$$

- consistent truncation to 3d: $g_{\mu\nu}\ ,\quad A_\mu^{(1,2)}\ ,\quad U_{1,2}$ — M.G., Strominger'10

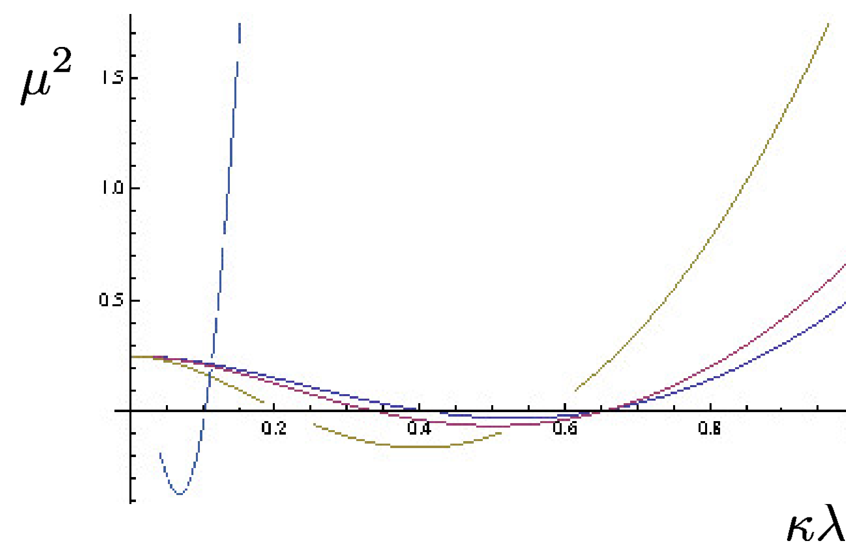
Chern-Simons

- warped black string solutions: λ, T_L, T_R — Detournay, MG '12

$$ds^2 = (1+\lambda^2 T_R^2)(ds^2_{BTZ} - A_\mu A_\nu) \qquad A_\mu^{(1,2)} = \alpha^{(1,2)}(\lambda T_R)\, A_\mu\ , \qquad U_{1,2}(\lambda T_R)$$

- Virasoro x Virasoro symmetry of non-propagating phase space

- propagating modes around black strings:



$$0 = (r^2 - (T_L T_R)^2)\, U_1''(r) + 2r\, U_1'(r) +$$

$$+ \left(\frac{1}{4} - \mu^2 + \frac{T_L^2\,\omega^2 + T_R^2\,\kappa^2 - 2r\omega\kappa}{4(r^2 - (T_L T_R)^2)}\right) U_1(r)$$

Stability analysis for travelling waves

- *global warped AdS* ($T_L^2 = -1$), *travelling waves* $\mu \in i\,\mathbb{R}$ → $\mu = i\eta$
- *solutions → Whittaker functions*
- *as* $r \to \infty$, *we have* $U \sim A\, r^{-\frac{1}{2}+i\eta} + B\, r^{-\frac{1}{2}-i\eta}$ *carry flux through boundary!*
- *zero flux condition:* $|A| = |B|$
- *regularity as* $r \to 0$

} *quantization condition on* ω

$e^{-i\,\omega\, u}$

  - *no instability found around vacuum* ($T_R = 0$)

Detournay, MG '12, Moroz '09

  - *instabilities around black hole solutions!* ($T_R \neq 0$)

Amsel, Horowitz, Marolf, Roberts '09

- *endpoint?*
- *different kinds of boundary conditions?*

Summary & future directions

- *toy models of warped AdS → Virasoro x Virasoro symmetry acting on pure gauge phase space*
- *extends to full (linearized) phase space when no travelling waves are present*
- *travelling waves → instability*

- *correct boundary conditions for travelling waves*
- *fate of the instability?*
- *extension of our results to the extreme Kerr black hole?*

Thank you!