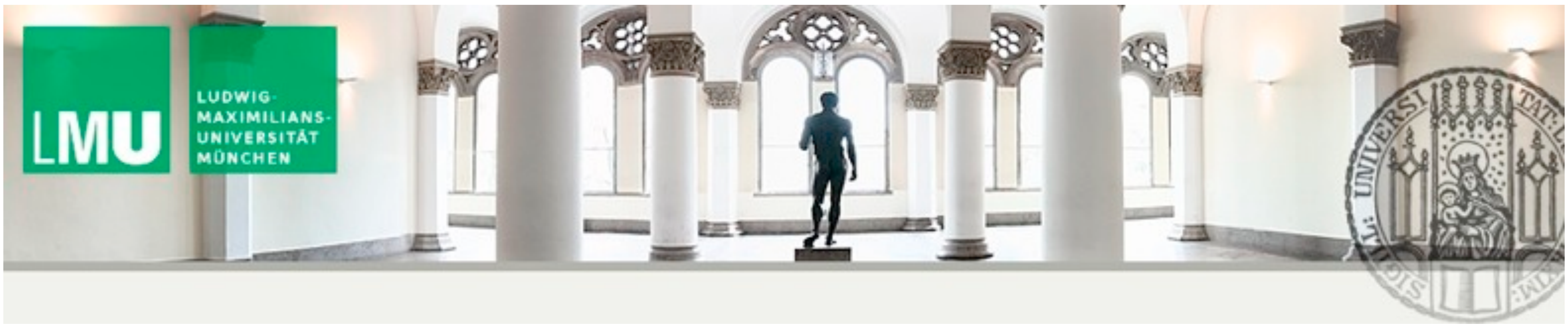
Classical & Quantum Black Hole Hair from Goldstone modes

DIETER LÜST (LMU-München, MPI)



Strings 2016, Beijing, August 1 - 5, 2016

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Work in collaboration with

Gia Dvali, Cesar Gomez, arXiv:1509.02114

Artem Averin, Gia Dvali, Cesar Gomez, arXiv:1601.03725, 1606.06260

Strings 2016, Beijing, August 1 - 5, 2016

Outline:

I) Introduction:

II) Classical Gravitational Black Hole Hair from Event Horizon Supertranslations

III) Quantum Hair & Charges from Black Holes as Graviton Boundstate

LIGO:

- Merging of binary black holes
- Discovery of gravitational waves

GW150914 GW151226

(Image credit: LIGO/A)

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- Conserved (asymptotic) charges of a black hole.
- The hair carries the information of collapsing matter that forms a black hole and subsequently evaporates via Hawking radiation.
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Quantum mechanically: one expects quantum hair.

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Our proposal: In addition to the BMS transformations at infinity there is a non-uniqueness of the classical black hole metric due to event horizon $\mathcal{A}$ - supertranslations:

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The $\mathcal{A}$ - supertranslations are spontaneously broken.

∞ many massless Goldstone modes of finite wave length $\Rightarrow$ infinite classical black hole entropy:

$$S_{class} = \infty \qquad\qquad Q_{class}^{\mathcal{A}} = 0$$

(ii) Quantum hair: non-uniqueness of finitely many almost degenerate black hole vacua:

- $\mathcal{A}$ - supertranslations generators generate transitions among the black hole vacua:

$$\mathcal{T}^{\mathcal{A}} \, |BH\rangle = |\widetilde{BH}\rangle$$

- $\mathcal{A}$ - supertranslations group will be explicitly broken.

- Finitely many charges which correspond to finitely many pseudo Goldstone modes of finite mass:

$$Q^{\mathcal{A}}_{q.m.} = \frac{1}{N} \neq 0 \ , \quad S_{q.m.} = N \Rightarrow$$ finite quantum hair.

(This is in accordance with the soft 1/N hair.)

[G. Dvali, C. Gomez, D.L. (2011)]

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Planck length: $L_P^2 \equiv \hbar G_N$

Entropy: $\mathcal{S} \sim N = \frac{r_S^2}{L_P^2}$

Classical limit:

$$\hbar \rightarrow 0\,,\, M = \text{finite},\;\; r_S = \text{finite}$$

Semiclassical limit:

$$\hbar = \text{finite},\;\; r_S = \text{finite},\;\; G_N \rightarrow 0,\;\; M \rightarrow \infty$$

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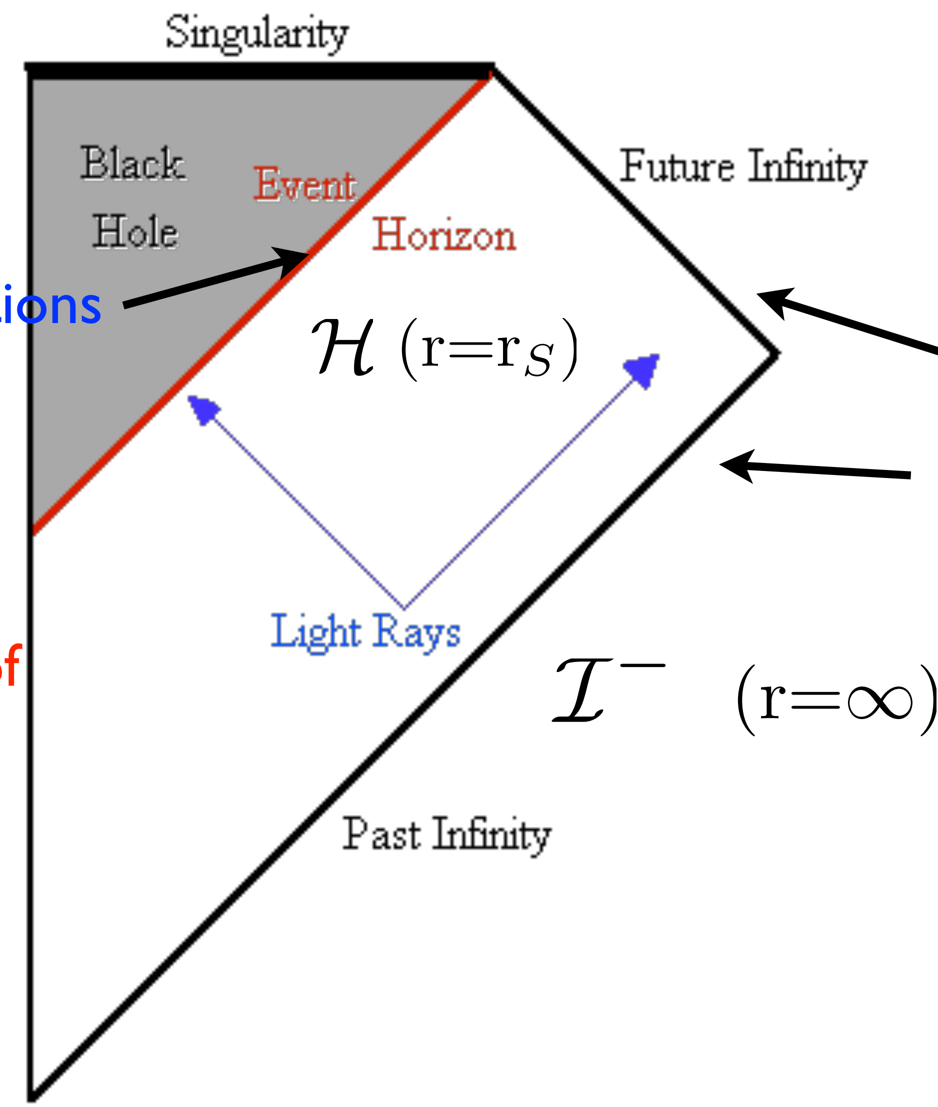
$\Big\}\ N \to \infty$

Consider Penrose diagramme for Schwarzschild metric:



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A - supertranslations

Goldstone modes of finite wave length

Standard BMS transformations

Goldstone modes of infinite wave length

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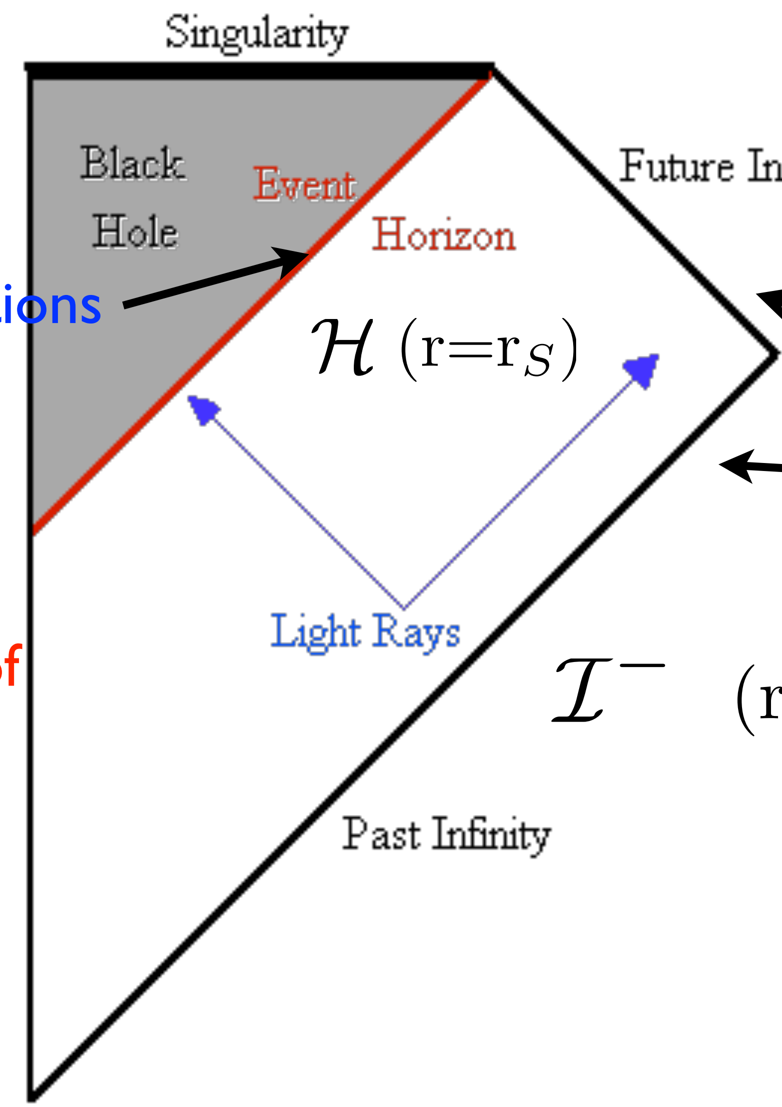

$\mathcal{H}$ (r=r_S)

$\mathcal{I}^+$ (r=∞)

$\mathcal{I}^-$ (r=∞)

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Standard BMS transformations

Goldstone modes of infinite wave length

Supertranslations at the horizon were also considered by

L. Donnay, G. Giribet, H. Gonzalez, M. Pino (2014); M. Blau, M. O'Loughlin (2015); H. Afshar, S. Detournay, D. Grumiller, W. Merbis, A. Perez, D. Tempo, R. Troncoso (2016); C. Eling, Y. Oz (2016); M. Setare, H. Adami (2016); G. Compere (2016); ....

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Diffeomorphisms that keep the boundaries (= null surfaces) of certain space-time metrics invariant.

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Bulk: gauge redundancies, no physical charges

Boundary: not gauge redundancies, but correspond to asymptotic symmetries, like large gauge transformations at infinity, which provide physical charges.

Schwarzschild metric in Edington - Finkelstein coordinates:

$$ds^2 = -(1 - \frac{r_S}{r})dv^2 + 2dvdr + r^2 d\Omega^2$$

$$v = t + r^* \qquad dr^* = (1 - \frac{r_S}{r})^{-1} dr$$

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(i) $r = \infty$: BMS transformations

$$ds^2 \ \rightarrow \ ds^2 + (\delta_{\eta_g} g_{\mu\nu}) \ r \ dx^\mu dx^\nu$$

(ii) $r = r_S$: $\mathcal{A}$ - supertranslations

$$ds^2 \ \rightarrow \ ds^2 + (\delta_{\chi_f} g_{\mu\nu}) \ r_S \ dx^\mu dx^\nu$$

(i) supertranslations on $\mathcal{I}^-$:

BMS: $$\delta_{\eta_g} g_{\mu\nu} = \begin{pmatrix} 0 & 0 & -\frac{1}{r}\frac{\partial g}{\partial \vartheta} & -\frac{1}{r}\frac{\partial g}{\partial \varphi} \\ 0 & 0 & 0 & 0 \\ * & * & 2\frac{\partial^2 g}{\partial \vartheta^2} & 2(\frac{\partial^2 g}{\partial \theta \partial \varphi} - \cot\vartheta \frac{\partial g}{\partial \varphi}) \\ * & * & * & 2(\frac{\partial^2 g}{\partial \varphi^2} + \sin\theta\cos\vartheta \frac{\partial g}{\partial \vartheta}) \end{pmatrix} .$$

Soft gravitons correspond to the Goldstone modes of BMS.

- Infinite wave length
- Infinite degeneracy of asymptotic flat spaces
- Infinite entropy but unaccessible information.

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- Infinite wave length
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They even act non-trivially on Minkowski space-time:

Infinite family of flat Minkowski metrics.

(ii) $\mathcal{A}$ - supertranslations at $\mathcal{H}$:

$$\mathcal{A}: \quad \delta_{\chi_f} g_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\frac{\partial^2 f}{\partial\vartheta^2} & -2(\frac{\partial^2 f}{\partial\vartheta\partial\varphi} - \cot\vartheta\frac{\partial f}{\partial\varphi}) \\ 0 & 0 & * & -2(\frac{\partial^2 f}{\partial\varphi^2} + \sin\theta\cos\vartheta\frac{\partial f}{\partial\vartheta}) \end{pmatrix}$$

- Gapless Goldstone (Bogoliubov) $\mathcal{A}$ - modes.

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- They correspond to gravitational „waves“ with finite wave length of order r_S , but zero energy (gap).

- They keep the ADM mass of the black hole invariant .
- They are solutions of the full non-linear Einstein equations on the horizon.

(A. Gußmann)

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These transformations generate an infinite dimensional algebra associated to infinitely many functions $\delta_{\chi_f} g_{\mu\nu}(\vartheta, \varphi)$:

$$\delta_{\chi_f} g_{\mu\nu}(\vartheta, \varphi) = \sum_{l=0}^{l=\infty} \sum_{m=-l}^{m=+l} b_{lm}^{\mu\nu} Y_{lm}(\vartheta, \varphi)$$

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- However all infinitely many classical $\mathcal{A}$ - supertranslation charges are zero:

$$\mathcal{Q}_{class}^{\mathcal{A}} \sim \int d^3x \; g^{\mu\nu} \partial_v (\delta_{\chi_f} g_{\mu\nu}) = 0$$

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Infinite entropy in (semi) classical limit: $S_{class} = \infty$

However it needs infinite time to resolve the information. Information is not accessible.

III) Quantum Hair & Charges from Black Holes as Graviton Boundstate (Coherent State Picture)

The $\mathcal{A}$ - supertranslation generators transform one black hole vacuum into another one:

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How does the quantum hair become finite?

How many angular Bogoliubov modes must be counted as information carriers?

Need microscopic quantum picture for black hole:

- Finite energy gap between the different black hole vacua.

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Bogoliubov excitations = pseudo Goldstone bosons.

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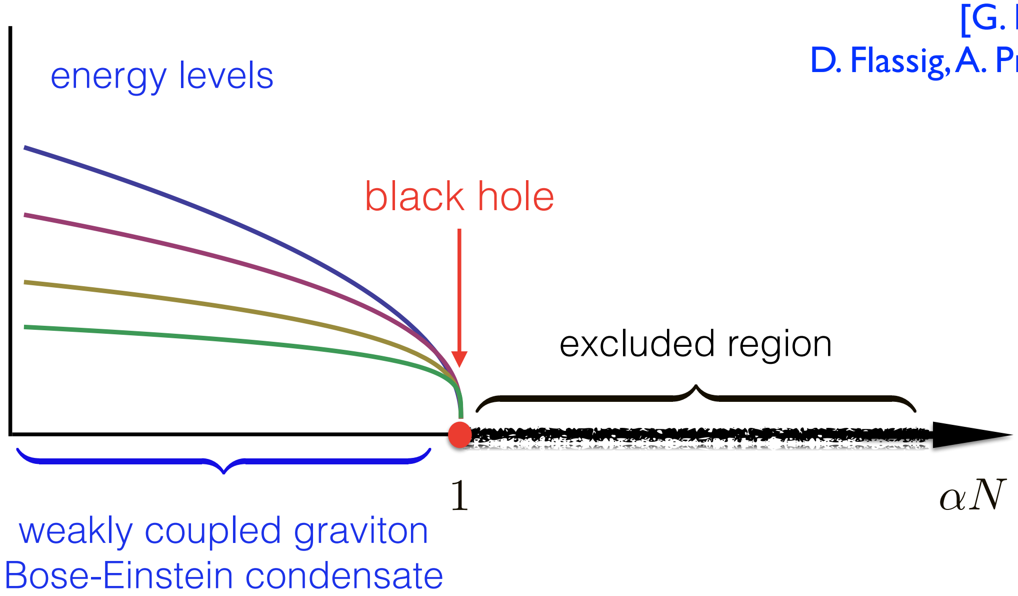
This agrees with the picture of a black hole as a bound state of N soft gravitons at a quantum critical point.

[G. Dvali, C. Gomez, 2011, G. Dvali, C. Gomez, D.L. (2012)]

Bogoliubov modes for black holes as BEC:

[G. Dvali, C. Gomez, D.L. (2012);
D. Flassig, A. Pritzel, N. Wintergerst, arXiv:1212.3344]



Quantum critical point at leading order in N:

$$\epsilon = \sqrt{1 - \alpha N} \longrightarrow 0$$

$$\alpha \equiv \frac{L_P^2}{R^2} \rightarrow \frac{1}{N}$$

For finite N subleading corrections: finite energy gap:

$$\omega_{lm} = \Delta E = \frac{1}{N}\frac{\hbar}{r_S}$$

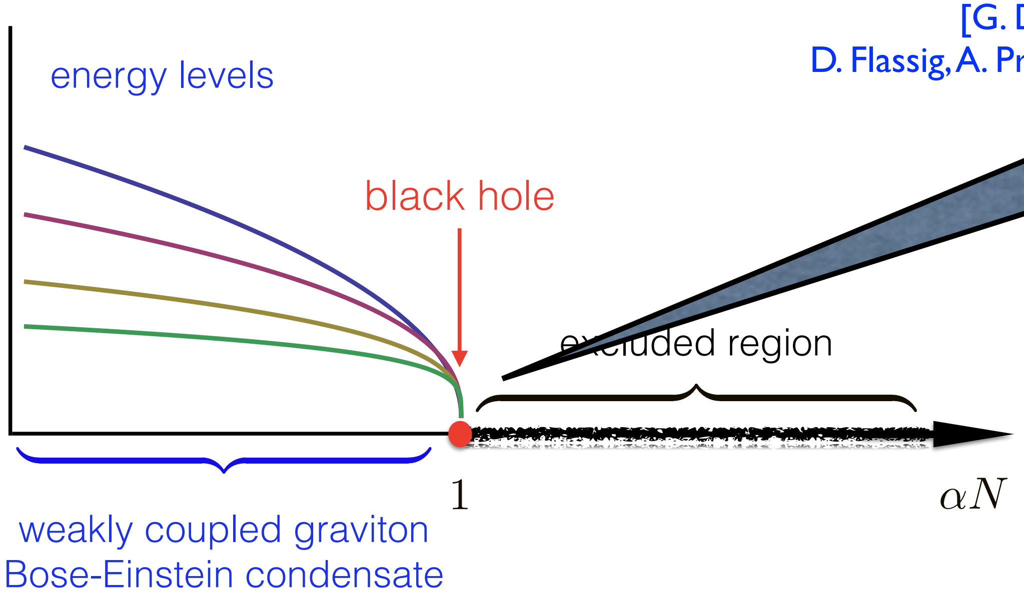
Bogoliubov modes for black holes as BEC:

[G. Dv

D. Flassig, A. Pritz

44]

Bogoliubov modes become (almost) gapless, degeneracy of states



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In quantum theory the $\mathcal{A}$ - modes correspond to operators of finite wave length gravitons:

$$\delta_{\chi_f}\hat{g}_{\mu\nu}(v,\vartheta,\varphi) = \sum_{l,m}\left(\hat{b}^{\mu\nu}_{lm}Y_{lm}(\vartheta,\varphi)e^{-iv\omega_{lm}} + \hat{b}^{\dagger,\mu\nu}_{lm}Y^{*}_{lm}(\vartheta,\varphi)e^{iv\omega_{lm}}\right)$$

Effective action on the horizon:

$$S_{eff} \sim \int dv d\vartheta d\varphi \Big(\partial_v\left(\delta_{\chi_f}\hat{g}_{\mu\nu}(v,\vartheta,\varphi)\right)\Big)^2$$

Charges:

$$\hat{Q}^{\mathcal{A}}_{lm} \sim \int g^{\mu\nu}\partial_v\left(\delta_{\chi_f}\hat{g}_{\mu\nu}(v,\vartheta,\varphi)\right) = -i\sqrt{\hbar\omega_{lm}}\left(e^{-i\omega_{lm}v}\hat{b}_{lm} - e^{i\omega_{lm}v}\hat{b}^{\dagger}_{lm}\right)$$

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Black hole vacua: coherent state of Bogoliubov modes:

$$|BH\rangle \equiv |N\rangle \sim e^{-\sum_{lm} \sqrt{n_{lm}} (\hat{b}_{lm} - \hat{b}^{\dagger}_{lm})} |0\rangle$$

Now it follows that

$$Q^A_{lm} = \langle N|\hat{Q}^A_{lm}|N\rangle \sim \sqrt{\hbar\omega_{lm} n_{lm}} \sim \frac{1}{N}$$

For finite N, the charges are non-vanishing, since the energy gap is non-zero:

$$\omega_{lm} \sim \Delta E \leq \frac{1}{N}$$

How many charges are there?

The energy gap can be estimated as $\Delta E \sim \frac{l^2}{N^2} \frac{\hbar}{r_S}$.

Restricting to weakly coupled modes $\Rightarrow \quad l_{max} \sim \sqrt{N}$

$-l \leq m \leq l \quad \Rightarrow$ There exist $l^2 = N$ different charges.

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There exist $l^2 = N$ different Boguliubov modes b_{lm} .

Each Bogoliubov qubit carries (at least) one bit of information.

$\Rightarrow$ The number of states is 2^N .

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Black hole qubits can be used as universal quantum computer.

[G. Dvali, M. Panchenko (2015); G. Dvali, C. Gomez, D.L., Y. Omer, B. Richter, 2016]

Final Remarks:

- Note that Minkowski space can be regarded as the near horizon limit of the Schwarzschild geometry, obtained in the limit $r_S \longrightarrow \infty$.

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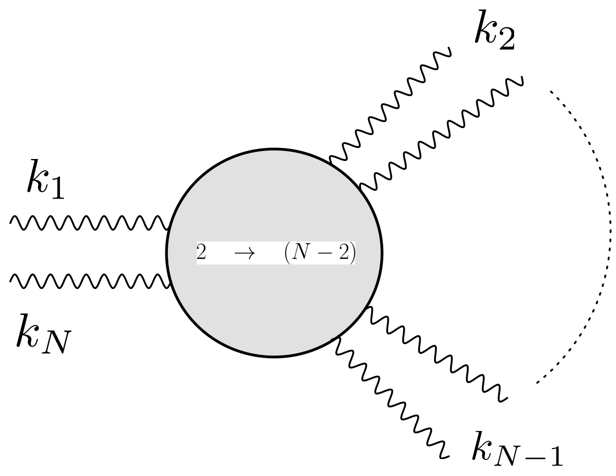
In quantum language it means that the corresponding Minkowski vacua are infinitely degenerate and can be regarded as coherent state of infinitely many gravitons with zero momentum:

[G. Dvali, C. Gomez, D.L., arXiv:1509.02114]

$$\mathcal{T}^{\mathcal{BMS}^-}\,|Min\rangle = |\widetilde{Min}\rangle$$



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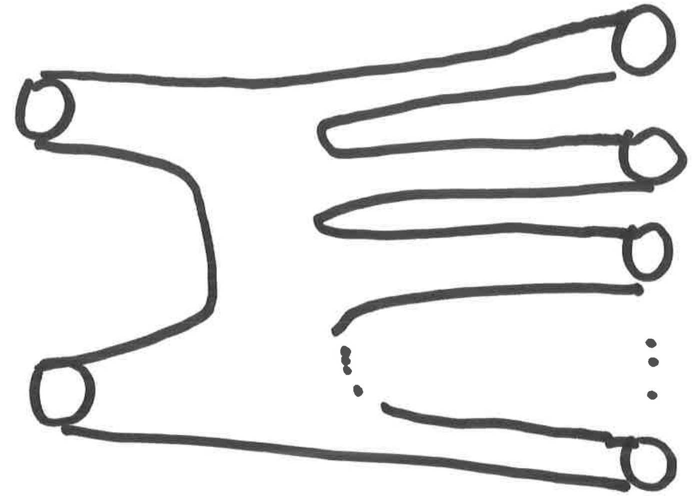

Field theory String theory

- S - matrix picture: 2 → N graviton scattering

  in special kinematical regime

[G. Dvali, C. Gomez, R. Isermann, D.L., S. Stieberger, 2014]

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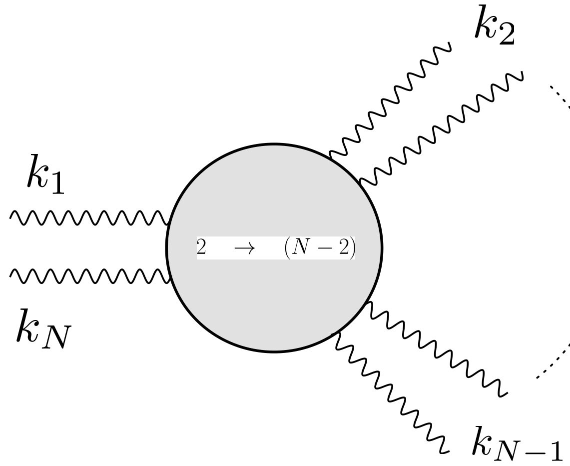

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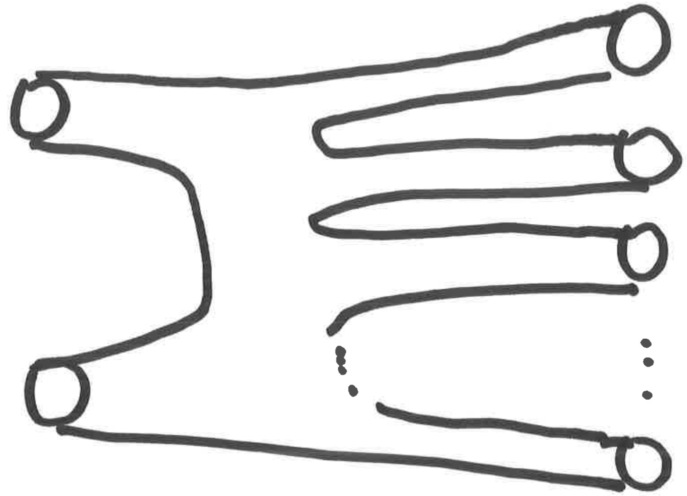

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Thank you very much !!

Back-up slides:

Supertranslations at the horizon $\mathcal{H}$:

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Boundary conditions:

The supertranslations should preserve the structure of the metric at the horizon:

$$f|_{r=r_S} = f(\theta,\phi)\,,$$

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Furthermore we require that $\delta_f g_{r\mu} = 0$

$\Rightarrow$ DGL‘s

$$\begin{aligned}
\frac{\partial f}{\partial r} &= 0\,, \\
\frac{\partial A}{\partial r} &= 0\,, \\
\frac{\partial}{\partial r}(r^2 B) + \frac{\partial f}{\partial \vartheta} - 2rB &= 0\,, \\
\frac{\partial}{\partial r}(r^2 \sin^2(\vartheta) C) + \frac{\partial f}{\partial \varphi} - 2r\sin^2(\vartheta) C &= 0.
\end{aligned}$$

Solutions:

$$BMS^{\mathcal{H}} \ : \quad \zeta^\mu_f = \Bigg(f(\vartheta,\varphi), 0, \frac{\partial f}{\partial \vartheta}(\frac{1}{r} - \frac{1}{r_S}), \frac{1}{\sin^2\vartheta}\frac{\partial f}{\partial \varphi}(\frac{1}{r} - \frac{1}{r_S})$$

Solutions:

$$BMS^{\mathcal{H}}\ :\quad \zeta_f^\mu = \Big(f(\vartheta,\varphi), 0, \frac{\partial f}{\partial \vartheta}(\frac{1}{r}-\frac{1}{r_S}), \frac{1}{\sin^2\vartheta}\frac{\partial f}{\partial\varphi}(\frac{1}{r}-\frac{1}{r_S})$$

Variation of metric:

$$\delta_{\zeta_f} g_{\mu\nu} = \begin{pmatrix} 0 & 0 & -(1-\frac{r_S}{r})\frac{\partial f}{\partial\vartheta} & -(1-\frac{r_S}{r})\frac{\partial f}{\partial\varphi} \\ 0 & 0 & 0 & 0 \\ * & * & 2r^2(\frac{1}{r}-\frac{1}{r_S})\frac{\partial^2 f}{\partial\vartheta^2} & 2r^2(\frac{1}{r}-\frac{1}{r_S})(\frac{\partial^2 f}{\partial\vartheta\partial\varphi}-\cot\vartheta\frac{\partial f}{\partial\varphi}) \\ * & * & * & 2r^2(\frac{1}{r}-\frac{1}{r_S})(\frac{\partial^2 f}{\partial\varphi^2}+\sin\vartheta\cos\vartheta\frac{\partial f}{\partial\vartheta}) \end{pmatrix}$$

(ii) Standard BMS supertranslations at $\mathcal{I}^-$:

Variation of black hole metric:

$$\delta_{\eta_g} g_{\mu\nu} = \begin{pmatrix} 0 & 0 & -(1-\frac{r_S}{r})\frac{\partial g}{\partial \vartheta} & -(1-\frac{r_S}{r})\frac{\partial g}{\partial \varphi} \\ 0 & 0 & 0 & 0 \\ * & * & 2r\frac{\partial^2 g}{\partial \vartheta^2} & 2r(\frac{\partial^2 g}{\partial \theta \partial \varphi} - \cot\vartheta \frac{\partial g}{\partial \varphi}) \\ * & * & * & 2r(\frac{\partial^2 g}{\partial \varphi^2} + \sin\theta\cos\vartheta\frac{\partial g}{\partial \vartheta}) \end{pmatrix} .$$

(iii) $\mathcal{A}$ - supertranslations: microstates of black hole:

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Compare supertranslations on $\mathcal{I}^-$ and on $\mathcal{H}$:

Horizon supertranslations contain a part which cannot be compensated by standard BMS-supertranslations.

Belong to the factor space: $\mathcal{A} \equiv BMS^{\mathcal{H}}/BMS^-$:

$$\begin{aligned} \chi^\mu_f &= \zeta^\mu_f - \eta^\mu_f \\ &= (0, 0, -\frac{1}{r_S}\frac{\partial f}{\partial \vartheta}, -\frac{1}{r_S \sin^2\vartheta}\frac{\partial f}{\partial \varphi}) \end{aligned}$$

These transformations are intrinsically due to the presence of the horizon.

The corresponding fluctuations of the metric correspond to physical massless modes of the black hole:

$$\delta_{\chi_f} g_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2r^2 \frac{1}{r_S} \frac{\partial^2 f}{\partial \vartheta^2} & -2r^2 \frac{1}{r_S} \left(\frac{\partial^2 f}{\partial \vartheta \partial \varphi} - \cot \vartheta \frac{\partial f}{\partial \varphi} \right) \\ 0 & 0 & * & -2r^2 \frac{1}{r_S} \left(\frac{\partial^2 f}{\partial \varphi^2} + \sin \theta \cos \vartheta \frac{\partial f}{\partial \vartheta} \right) \end{pmatrix}$$