DFT
A Pathway to Quantum Strings

Soo-Jong Rey

B.W. Lee Center of Fields, Gravity & Strings

Institute for Basic Science, Korea

I am NOT going to talk about higher-spin theory.

- colored higher-spin gauge theory
- partially massless Higgs mechanism

Rather, I would like to share with you thoughts on something ”explorative” and ”wild”.

- ArXiv 1507.07545
- ArXiv 1510.06735
- Work in progress 1608.nnnnn
- In collaboration with
  Kanghoon Lee, Jeong-Hyuck Park, Woohyun Rim (Seoul)
  Yuho Sakatani (Kyoto), Alejandro Rosabal (Buenos Aires)

ibs
Institute for Basic Science
a research institute of scientist's dreams

cf. talks by Arkani-Hamed, Yu-Tin Huang

A Student Project (I): Calculable & Predictable?

$$
\begin{aligned}
I \;=\; & \int \mathrm{d}^4 x \, \Big[\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\psi)^2 + \frac{1}{2} g^2 \phi^4 (\partial\phi)^2 + \frac{1}{2}\lambda^2 \psi^4 (\partial\psi)^2 \\
+ \; & \frac{1}{2}(\ln 2)^2 \kappa^2 \left(\frac{1}{7}\phi^4\psi^7(\partial\phi) + \frac{1}{5}\phi^5\psi^6(\partial\psi)\right)^2 + \frac{1}{2}\zeta^2(7) h^2 \psi^{16}(\partial\psi)^2 \\
+ \; & \frac{1}{2}(\ln 3)^2 \eta^2 \left(\frac{1}{9}\phi^9\psi^4(\partial\psi) + \frac{1}{5}\phi^8\psi^5(\partial\phi)\right)^2 + \frac{1}{2}\zeta^2(4) \sigma^2 \phi^{14}(\partial\phi)^2 \\
+ \; & \frac{1}{3} g (\partial\phi)(\partial\phi^3) + \frac{1}{2}\lambda(\partial\psi)(\partial\psi^2) + [\zeta(7) h \psi^8 + \zeta(4)\sigma\phi^7](\partial\phi\partial\psi) \\
+ \; & \frac{\ln 2}{35}\kappa(\partial\phi)\partial(\phi^5\psi^7) + \frac{\zeta(7)}{27} g h \partial(\phi^3)\partial(\psi^9) + \frac{\ln 2}{105} g\kappa \partial(\phi^3)\partial(\phi^5\psi^7) \\
+ \; & \frac{\ln 3}{45}\eta(\partial\psi)\partial(\phi^9\psi^5) + \frac{\zeta(4)}{16}\lambda\sigma\partial(\psi^2)\partial(\phi^8) + \frac{\ln 3}{90}\lambda\eta\partial(\psi^2)\partial(\psi^5\phi^9) \\
+ \; & \frac{\ln 2\,\zeta(7)}{315} h\kappa \partial(\phi^9)\partial(\phi^5\psi^7) + \frac{\ln 3\,\zeta(4)}{360}\sigma\eta\partial(\phi^8)\partial(\phi^9\psi^5) \\
+ \; & \text{(add extra if needed)}
\end{aligned}
$$

Secret code to Einstein gravity

$$
\begin{aligned}
I_{\text{gravity}} &= \frac{1}{16\pi G_N} \int \mathrm{d}^4 x \sqrt{-g} R(g) + (\text{Gibbons-Hawking}) \\
&= \int \mathrm{d}^4 x \left[(\partial h)^2 + h^2 (\partial h)^2 + h^3 (\partial h)^2 + \cdots \right]
\end{aligned}
$$

In particular,

Divergence at ℓ-loop = $\ell(d-2)+2$,

independent of the number of external lines.

After canonical transformation

$$\begin{aligned} \Phi &= \phi + \frac{g}{3}\phi^3 + \frac{\zeta(7)}{9} h\psi^9 + \frac{\ln 2}{35}\kappa\phi^5\psi^7 \\ \Psi &= \psi + \frac{\lambda}{2}\psi^2 + \frac{\zeta(4)}{8}\sigma\phi^8 + \frac{\ln 3}{45}\eta\phi^9\psi^5, \end{aligned}$$

the theory is **FREE**:

$$I_{\text{student}} = \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}(\partial\Psi)^2$$

Ward identities:

$$\frac{Z_{\phi^3}}{Z_\phi} = \frac{Z_{\psi^9}}{Z_{\phi^3}} = \frac{Z_{\phi^5\psi^7}}{Z_{\psi^9}} = 1, \qquad \frac{Z_{\psi^2}}{Z_\psi} = \frac{Z_{\phi^8}}{Z_{\psi^2}} = \frac{Z_{\phi^9\psi^5}}{Z_{\phi^8}} = 1$$

[5]

Student Project (II): Calculable and Predictable?

O(N) nonlinear sigma model:

$$I = \int \mathrm{d}^4x \sum_{a,b=1}^{N-1} G_{ab}(\phi)\partial\phi^a\partial\phi^b, \qquad G_{ab} = \delta_{ab} + \frac{\phi_a\phi_b}{F^2 - \phi^2}$$

The model has two-derivative, non-polynomial interactions. At tree level, it behaves badly at real energy scales above $\sim 4\pi F$. At loop level, it behaves badly at virtuality above $\sim 4\pi F$.

To get better UV behavior, one must append the model with additional degrees of freedom at the threshold scale $\sim 4\pi F$. This UV completion is done by adding a Higgs field σ as the N-th component, replacing the scale $4\pi F$:

$$I = \int \mathrm{d}^4x \sum_{A=1}^{N} \partial\Phi_A\partial\Phi_A + \frac{\lambda}{4}\left(\sum_{A=1}^{N} \Phi_A\Phi_A - F^2/\lambda\right)^2$$

Maxim from Student Project

To attain better high-energy behavior and renormalizability, calculability and hence predictability of a theory framework:

Enlarge field degrees of freedom

Choose Variables Smartly

Enlarge gauge/global symmetries

Quantum Theory of Gravity

- string theory constructed as a theory of quantum gravity
- Prohibitively complicated, so study by ”divide & conquer”
- couplings, kinematical asymptotics, gauge symmetries

Starting from Einstein gravity, take pathway to string theory via

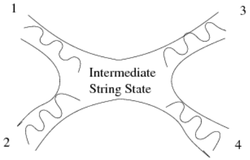
(1) (g, b, ϕ) supergravity = infinite tension, low-energy limit

(2) $(g_2, g_3, \cdots)$ higher-spin = zero tension, high-energy limit

(3) double field theory = enlarged gauge symmetry limit

Remark on Massive Higher Spins and Strings

- Causality constraint $\rightarrow$ a tower of massive higher spins
  Maldacena et.al.
- Is the tower of massive higher spins to be automatically equated to string theory?



- Consider AdS$_{d+1}$ Vasiliev theory compactified to AdS$_d \otimes$ Janus geometry with two boundaries Bak, Gutperle, Hirano
- infinite Kaluza-Klein tower of massive higher spins in AdS$_d$
  Gwak, Kim, SJR
- Shapiro time delay in AdS$_d$ necessitates higher spins
- At UV, they come from AdS$_{d+1}$ Vasiliev, not from string (yet)

Roadmap for Higgs Hunt

String Theory

$\Uparrow$

A Pathway ?

$\Uparrow$

Einstein Gravity

Pathway through Double Field Theory (DFT)

Einstein gravity	$\subset$ Double Field Theory	$\subset$ String Theory
$\mathit{Diff}(\mathrm{M}_D)$	$\subset$ $\mathit{Diff}(\mathrm{M}_{2D})$	$\subset$ $G_{\rm string}$

[Question]

How (much) is high-energy behavior improved?

[Note] DFT as a proxy for the MAXIM

DFT - Spacetime Description

Aldazabal, Berman, Blumenhagen, Grana, Gwak, Hohm, Hull, Jeon, Lee, Lüst, Nunez, Park, Sen, Siegel, Waldram, Zwiebach

- start with T-duality symmetry of strings Kikkawa, Yamazaki, Narain
- treat momentum, winding on equal footing: $(x^m, \tilde{x}_m) := X^M$

$$\mathcal{M}_D \otimes \tilde{\mathcal{M}}_D \longrightarrow \hat{\mathcal{M}}_{(D,D)} \longrightarrow \overline{\mathcal{M}}_D / O(D,D,\mathbb{Z})$$

$$\partial_A \partial^A \Phi(X) \simeq 0, \qquad \partial_A \Phi(X) \partial^A \Psi(X) \simeq 0.$$

- $(g, b, \phi)_D \rightarrow (g, b, \phi)_{(D,D)}$, then project down to $\overline{\mathcal{M}}_D$
- combine Diff(g) and G(b) with O(D,D) covariance

$$G_{\rm DFT} = Diff(\hat{\mathcal{M}}_{(D,D)}); \qquad Diff(\hat{\mathcal{M}}_{(D,D)}) \gg Diff(\mathcal{M}_D)$$

- DFT endows enormously enlarged gauge invariance to the massless string modes (g, b, ϕ), at the apparent expense of manifest locality

DFT – Worldsheet Description

- $x(z,\bar{z}) = x_L(z) + x_R(\bar{z})$; $\tilde{x}(z,\bar{z}) = x_L(z) - x_R(\bar{z})$
- T-dualty

$$x_L \to x_L, \qquad x_R \to -x_R$$

  is the origin of O(D, D) signature of $X^M = (x, \tilde{x})$
- Level-matching condition leads to the section constraints

$$(L_0 - \bar{L}_0)|\Phi\rangle = 0 \quad \to \quad \partial^2\Phi = 0, \quad \partial_A\Phi\partial^A\Psi = 0$$

- $G_{\mathrm{DFT}} = \mathit{Diff}(\hat{\mathcal{M}}_{(D,D)})$ arises from closed SFT gauge algebra, which receives α'-corrections after $O(D,D)$ non-covariant field and parameter redefinitions

DFT at Leading Order in α'

- Fields $(g, b, \phi) \to (\mathcal{M}_{MN} = \mathcal{E}_M \cdot \mathcal{E}_N, d)$ smart choice of variables
- O(D,D) invariant metric and O(D,D) covariant background

$$\mathcal{J} = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}; \qquad \mathcal{H} =< \mathcal{M} >$$

- At leading-order in α', $\mathcal{M}$ is constrained cf. O(N) nonlinear sigma model

$$\mathcal{M}^2 = \mathcal{J}; \qquad \mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}b \\ bg^{-1} & g - bg^{-1}b \end{pmatrix}$$

$$\begin{aligned} S_0 &= \int e^{-2d}\mathcal{R}(\mathcal{E}) + \oint e^{-2d}\mathcal{L}_{\mathrm{GH}}(\mathcal{E}) \quad \text{subject to} \quad \mathcal{M}^2 = \mathcal{J} \\ \mathcal{R} &:= \mathcal{M}\partial\mathcal{M}\partial\mathcal{M} + \partial\partial\mathcal{M} + \mathcal{M}\partial d\partial d + \partial\mathcal{M}\partial d + \partial\mathcal{E}\partial\mathcal{E}\mathcal{M} \end{aligned}$$

- The action is uniquely fixed by G_{DFT}! enlarged gauge symmetry
- Unfortunately, the constraint $\mathcal{M}^2 = \mathcal{J}$, puts the weak field expansion of $\mathcal{M}$ as $g = \eta + h$ and $g^{-1} = \eta + h + h^2 + \cdots$ non-polynomial.

At leading-order in α', the DFT achieves enormously enlarged gauge symmetry G_{DFT}, but the field variables are still not smart

DFT at leading order in $\alpha' \simeq$ Einstein gravity

DFT at Next Order in α'

- Metaphor: Nonlinear to Linear O(N) model with Higgs
- At next order, DFT miraculously manages to achieve this:

$$
\begin{aligned}
S_1 &= \int e^{-2D}\Big[(\mathcal{M} - \frac{1}{3}\mathcal{M}^3) \\
&+ \alpha'((\mathcal{M}^2 - 1)\mathcal{M}\partial\partial D + \mathcal{M}\partial\mathcal{M}\partial\mathcal{M} + \mathcal{M}\partial\partial D) \\
&+ O(\alpha'^2\partial^4) + O(\alpha'^3\partial^6)\Big]
\end{aligned}
$$

- Enlarged gauge symmetry remains the same, G_{DFT}
- $\mathcal{M}$ is no longer constrained; $\mathcal{S}_1$ is polynomial in fields
- The action is uniquely determined by G_{DFT}
- No other possible counter-terms up to this order

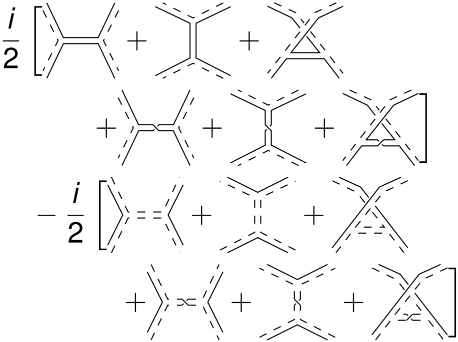
- What are the "Higgs" modes?
- Is high-energy behavior improved?

Higgs modes?

- $\mathcal{M}$ is no longer constrained
- S_0 contains ∂^2 terms, S_1 contains up to ∂^6 terms.
- $\mathcal{M} \simeq (g, b, \phi) \oplus (m, \bar{m})_{(mn)}$ from weak field expansion
- By O(D,D) field redefinition, only ∂^2 terms are relevant

$$\alpha' \Box m - 4m = \mathcal{L}(\mathcal{M}, m, \bar{m}) = h^T h + \cdots$$
$$\alpha' \Box \bar{m} + 4\bar{m} = \bar{\mathcal{L}}(\mathcal{M}, m, \bar{m}) = h\, h^T + \cdots$$

- It can be viewed as quiver matrix theory associated with double spin-Lorentz symmetry O(D)⊗O(D). Arkani-Hamed+Kaplan
The $m, \bar{m}$ fields are symmetric, the (h, b) is bi-fundamental.
It admits large-D expansion. cf. Strominger, Emparan, Minwalla, higher-spin theory



Higgs Modes?

Extra fields $(m, \bar{m})$ are the "Higgs" fields with features:

- $\mathcal{M}$ is no longer constrained; extra DOFs = $m, \bar{m}$
- $m_{0i}, \bar{m}_{0i}$ are non-dynamical fields cf. not Lagrage multiplier
- negative norm, akin to Pauli-Villar and Lee-Wick

  Boulware+Gross
- $m^2 = \pm 4/\alpha'$; this precise massive pair is needed upon integrating out $m, \bar{m}$ to cancel ∂^0 terms for $\mathcal{M}$ absent in S_0

  cf. Hohm, Nasser, Zwiebach

High-Energy Behavior (I)

- 4-point (h, b) amplitudes

cf. Huag, Siegel, Yuan; Huang's talk

$$A_4^{\rm DFT}(s,t,u) \sim \left(1 + \frac{su}{s^2 - 4/\alpha'^2} + \cdots\right) A_4^{\rm grav}(s,t,u)$$

- KLT kernel

$$A_4^{\rm DFT} = A_4^{\rm chiral}(+\eta) K_{\rm DFT} \bar{A}_4^{\rm chiral}(-\eta)$$

where

$$K_{\rm DFT} = \left(1 + \frac{su}{s^2 - 4/\alpha'^2} + \cdots\right) K_{\rm grav}$$

- For such soft UV behavior, both ”negative-norms” and ”precise mass-squared spectrum $\alpha' m^2 = \pm 4$ are crucial

High Energy Behavior (II)

- BCFW factorization viewed as soft-collinear scattering

  Arkani-Hamed+Kaplan
- At large z, enhanced spin symmetry governs the leading behavior
- For leading DFT, the same as Einstein gravity cf. Boels+Hurst
- For $O(\alpha')$ DFT, BCFT asymptotics gets more convergent

$$A_4^{\mathrm{DFT}}(-,-;\pm,\pm) \to \underbrace{\left(\frac{1}{z} + \cdots\right)}_{\mathrm{DFT}} \cdot \frac{1}{z^s}\Big|_{s=2}$$

- Similar softer behavior for other polarizations

High Energy Behavior (III)

- vacuum amplitudes

cf. bosonic YM theory at d=26; Tseytlin, SJR

$$
\begin{aligned}
A_0 &= \int \mathrm{d}^4 p \left[\sum_{(h,b,d)} \log p^2 - \sum_{m,\bar{m}} \log(p^2 \pm 4) \right] \\
&\simeq \left[(d-2)^2 - 2 \cdot \frac{1}{2}(d-1)^2 + 2(d-1) + 1 \right] \Lambda^4 + \cdots
\end{aligned}
$$

- Subleading divergence uncancelled
- Radiative correction to Newton's constant non-vanishing from leading order

At higher-order in α', the DFT retained enormously enlarged gauge symmetry G_{DFT}, and also the "Higgs" modes that soften the high-energy behavior

DFT at higher order in $\alpha' \simeq$ UV improved gravity

cf. Higher-spin theory at higher loop order

Giombi, Klebanov, Tseytlin, Beccaria, ...

Remark on Indefinite Hilbert Space

- $m, \bar{m}$ are ghosts
- dynamical Pauli-Villar fields and Lee-Wick mechanism?

  Boulware, Gross; Grinstein, Wise
- If arising from a certain limit of string theory, then "how"?
- Could they be viewed as "effective" description of contribution of infinitely many positive-norm states?

$$2\sum_{s=1}^{\infty}\frac{+1}{p^2 \pm m^2} = \frac{2\zeta(0)}{p^2 \pm m^2} = \frac{-1}{p^2 \pm m^2}$$

Remarks for Ambitwistor Strings

- $O(\alpha')$ DFT was derived from chiral CFT and hence "chiral string" dynamics Hohm, Siegel, Zwiebach
- We derived this chiral string from conventional string after integrating out anti-chiral part and taking infinite tension limit; This fits nicely on "how" the spacetime signature changes between original and T-dual coordinates in DFT coordinates $X^M = (x, \tilde{x})$ see also Hai-Tang Yang

$$\langle x^m(z,\bar{z})\tilde{x}^n(z,\bar{z})\rangle = \eta^{mn}\log\frac{z}{\bar{z}} = +\eta^{mn}\log z - \eta^{mn}\log\bar{z}$$

- The signature change converts the standard KLT to closed string amplitude to DFT amplitude Huang, Siegel, Yuan
- Ambitwistor string approach to YM and gravity scattering amplitudes and explanation of CHY scattering equation

cf. Cachazo, He, Yuan; Casali, Tourkine

Remark on Little DFT

- LST enjoys T-duality symmetry Vafa et.al.; Hohenegger, Iqbal, SJR; Kim, Kim, Lee
- no ten-dimensional (g, b, ϕ)
- 5-brane worldvolume fields (a, b^+) + massive excitations
- manifest O(D,D) covariant description of (a, b^+) leads to doubled gauge theory with enlarged gauge symmetries

little DFT = double gauge theory

$$S_{\text{littleDFT}} = \int e^{-2d} \frac{1}{2} \text{Tr} \mathcal{F}^2(A), \qquad A_M \simeq (a, b^+)$$

- As a pathway for UV completing gauge theories to LST, explore high-energy behavior of the little DFT
- Expect to shed light to noncritical / QCD strings cf. Komargodski
- Tension between massive higher-spins from QCD (a) versus from abelian Higgs model (b^+)?

Thank You

For out of olde feldes, aas men seith,
Cometh al this newe corn fro yeer to yere;
And out of olde bokes, in good feith,
Cometh al this newe science that men lere.

Geoffrey Chaucer