Bootstrap approach to CFT in D dimensions

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Strings 2013, Seoul

Origins of Conformal Bootstrap, early 1970's

Raoul Gatto

Sergio Ferrara

Aurelio Grillo

Alexander Polyakov

Results from those early days

- primary operators + descendants [Mack, Salam 1969]
- unitarity bounds [Ferrara, Gatto, Grillo 1974, Mack 1977]
- conformally invariant OPE
- constraints on the correlation functions of primaries

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They realized that:

1) Any CFT is characterized by

conformal data = {primary operator dimensions Δ_i, OPE coefficients c_{ijk}}

2) OPE associativity:

$$\langle (O_i O_j)(O_k O_l) \rangle = \langle (O_j O_k)(O_i O_l) \rangle \qquad \forall i, j, k, l$$

should fix the data $\Rightarrow$ conformal bootstrap

Enter QCD...

Conformal blocks

D>2 discl...

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta_\phi} x_{34}^{2\Delta_\phi}}$$

$$g(u,v) = 1 + \sum_i |c_{\phi\phi i}|^2 g_{O_i}(u,v)$$

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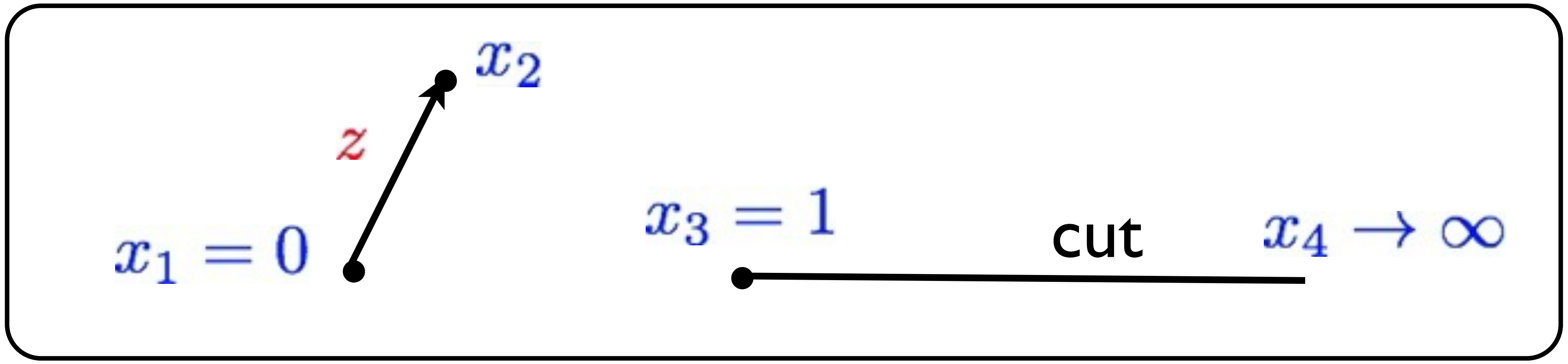
conf. blocks

$$\phi \times \phi = \mathbf{1} + \sum c_{\phi\phi i}(O_i + \text{descendants})$$

...and coordinates for them

z-coord:



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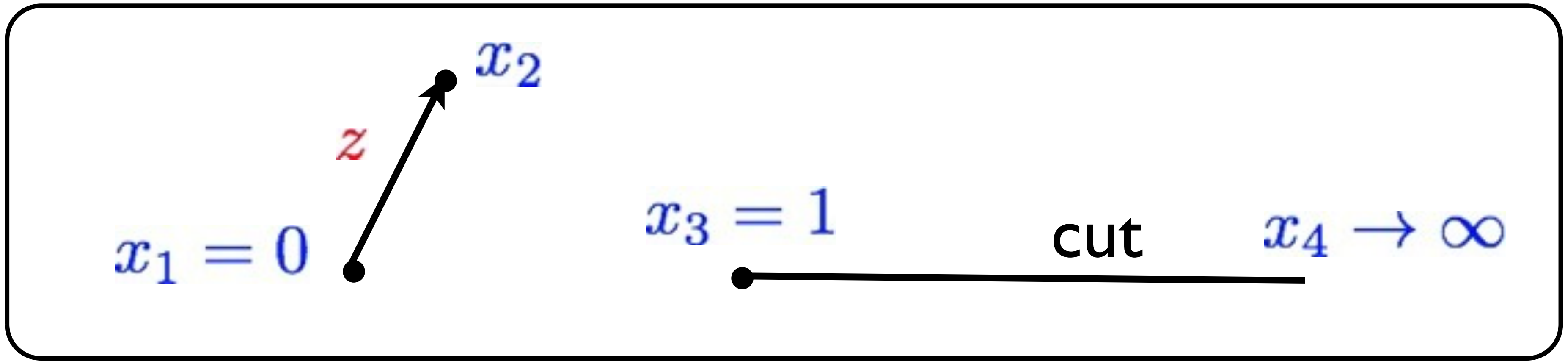

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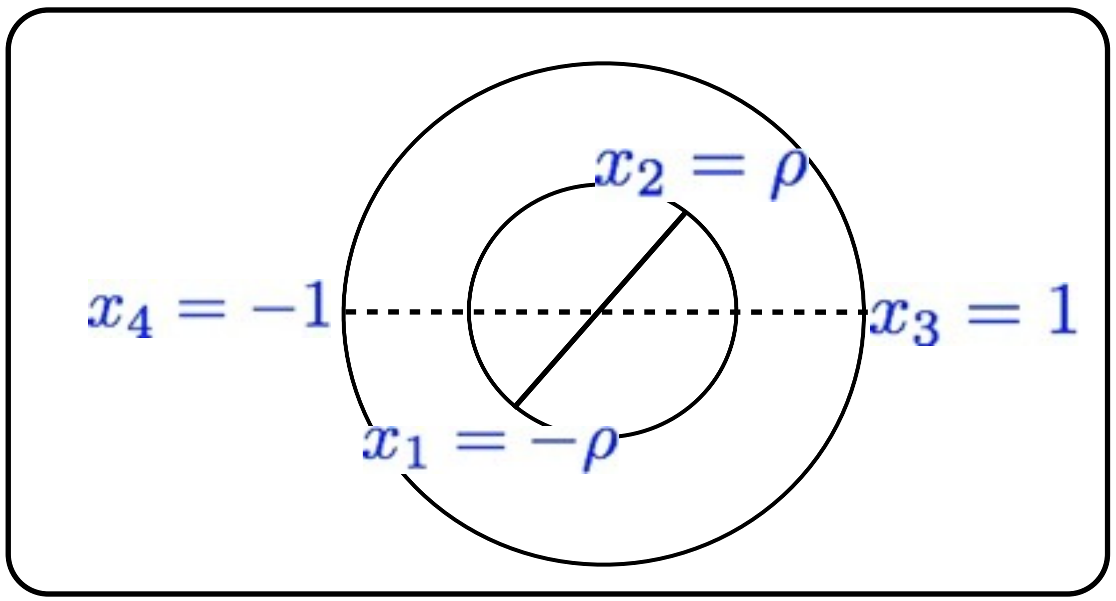

[Pappadopulo, S.R., Espin, Rattazzi 2012, Hogervorst, S.R. 2013]

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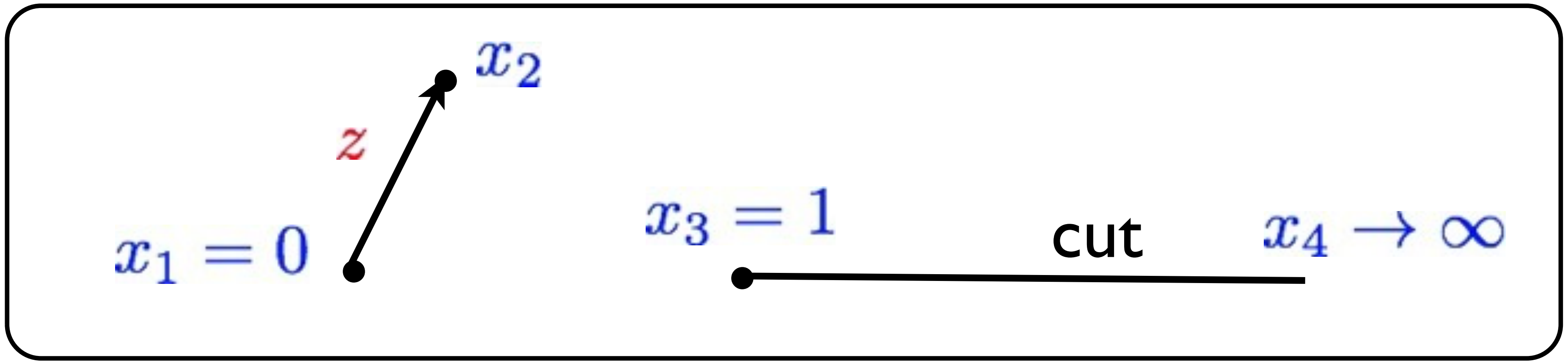

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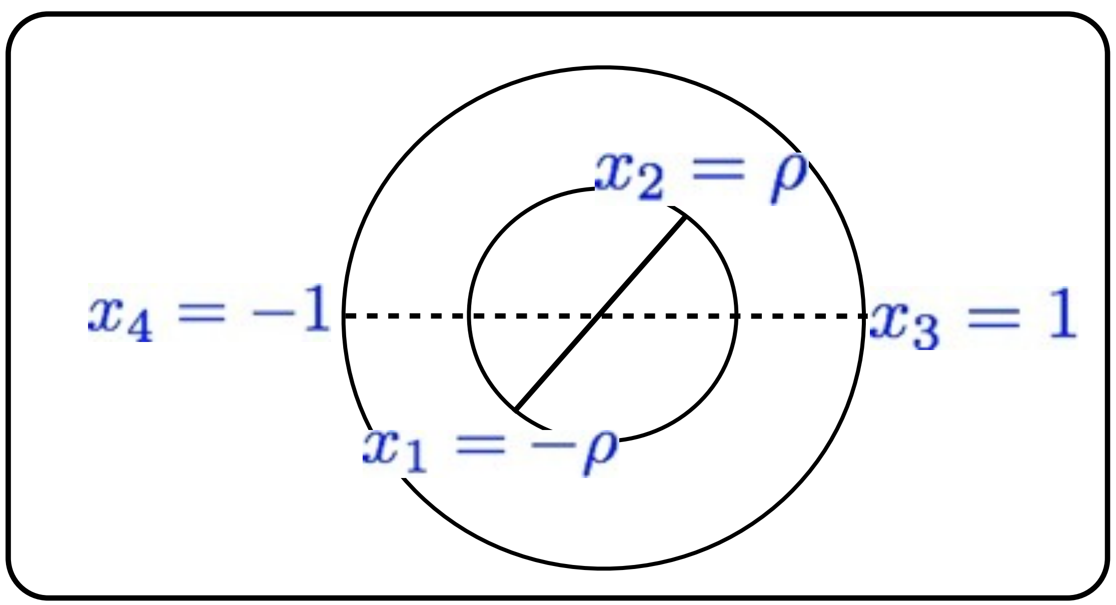

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$$\rho = \frac{z}{(1+\sqrt{1-z})^2}$$

$$g_{O_i}(\rho = re^{i\theta}) = \sum d_k r^{\Delta_i + k} \times \mathrm{Geg}_{l_k}^{(D/2-1)}(\cos\theta)$$

known coeffs.

D=2, Al.Zamolodchikov, fractional...

Convergence of conf. block decomposition

[Pappadopulo, S.R., Espin, Rattazzi 2012]

$$g(u,v) = 1 + \sum_i |c_{\phi\phi i}|^2 g_{O_i}(u,v)$$

$$\sim \frac{1}{(1-r)^{2\Delta\phi}} \times \frac{1}{(1-r)^{2\Delta\phi}} \qquad (r \to 1)$$

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Cf. $\sum \delta(E - \Delta_i) \sim \exp(\# E^{1-1/D})$

Simplest bootstrap equation

$$g(u,v) = 1 + \sum_i |c_{\phi\phi i}|^2 g_{O_i}(u,v)$$

crossing:

$(z \to 1 - z)$

$$u^{\Delta_\phi} g(v,u) = v^{\Delta_\phi} g(u,v)$$

Mathematically well-def'd, S-matrix...

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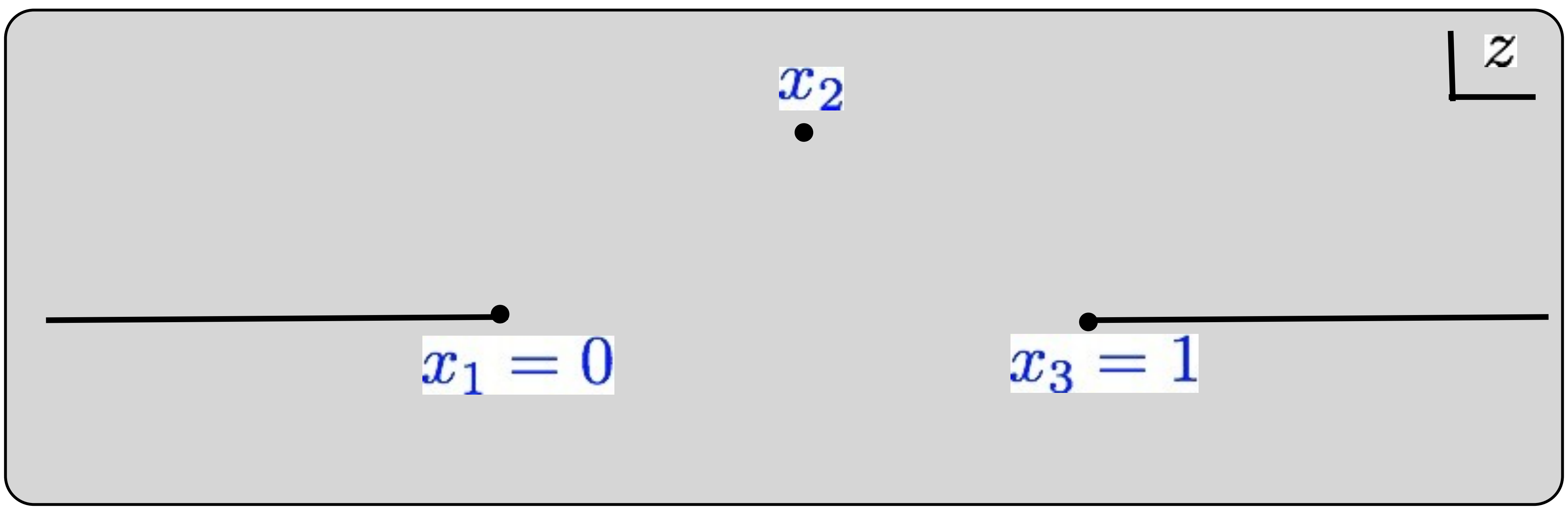

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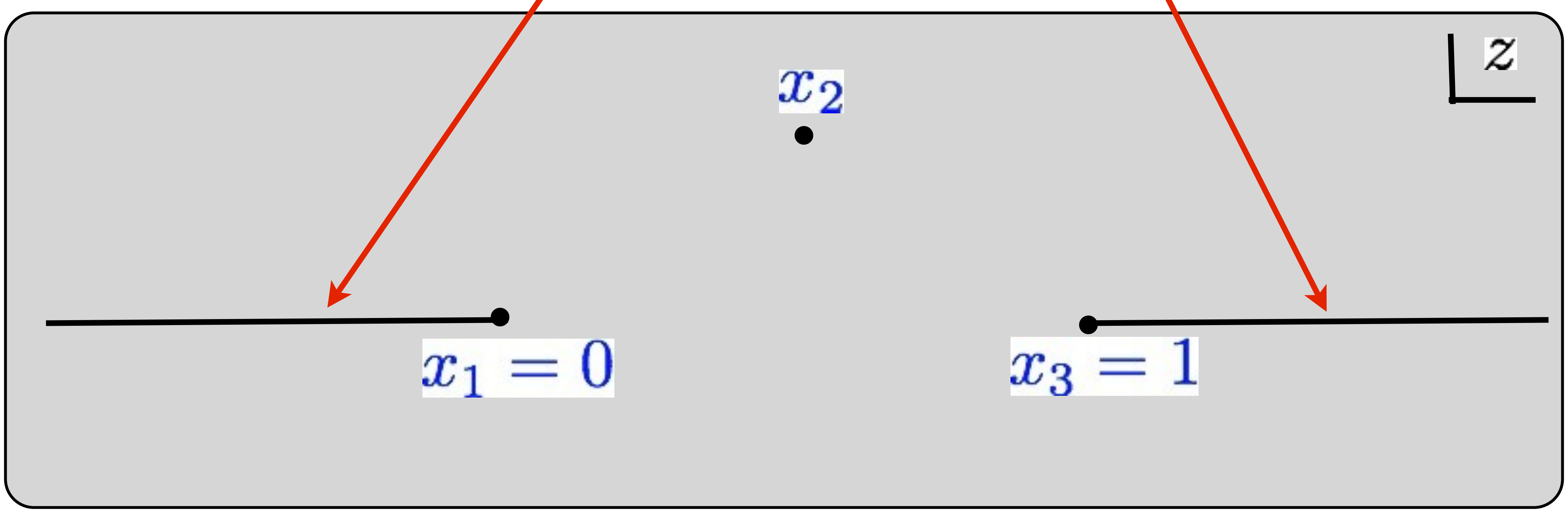
convergence cuts



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Numerical exploration

1) Identifying "swampland" in the space of CFT data

2) Study of theories at the "swampland boundary"

I. Charting out CFT "swampland"

[Rattazzi, S.R, Tonni, Vichi, 2008] + many subsequent works

Rule out large chunks of CFT data space
which do not correspond to any CFT,
because bootstrap equations do not allow a solution

Keyword: linear programming (way to enforce $p_i = |c_{\sigma\sigma i}|^2 \geq 0$)

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Roads to swampland:

increase gaps in the spectrum

pump up OPE coefficients

Example of a gap study

Take any CFT with G ⊃ SO(N) global symmetry

$$\sigma_a \times \sigma_b = \delta_{ab}(\mathbf{1} + S + \ldots) \quad + T_{ab} + \ldots$$
$$+(\ell \geq 1)$$

fund. of SO(N)

lowest dimension singlet and $\square\square$

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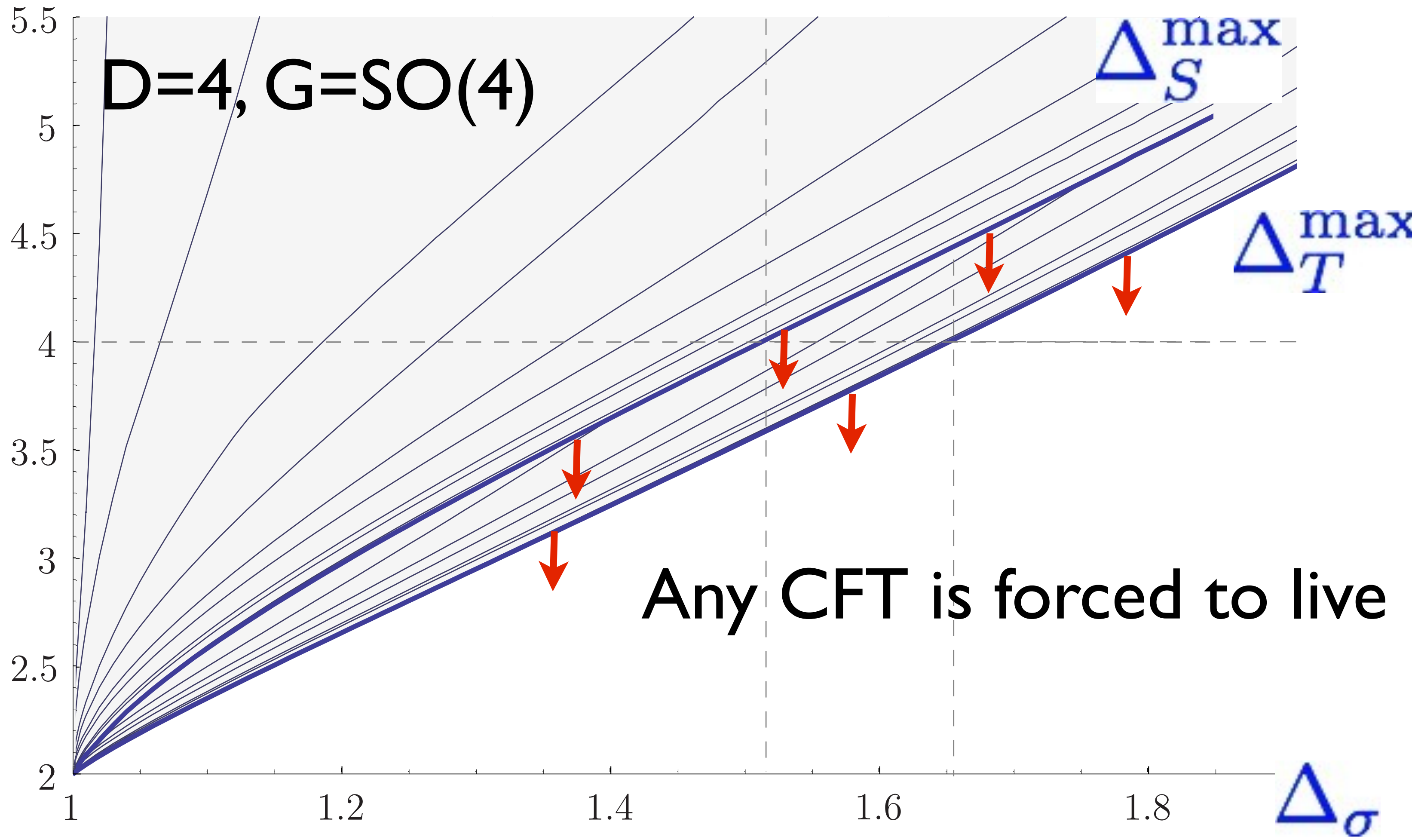

Any CFT is forced to live below these lines

[Vichi 2011, Poland, Simmons-Duffin, Vichi, 2011]

Analytic bounds from "toy bootstrap": [Hogervorst, S.R. 2013]

A central charge lower bound

Suppose know Δ_σ,

can we say something about $C_T \propto \langle T_{\mu\nu} T_{\mu\nu} \rangle$?

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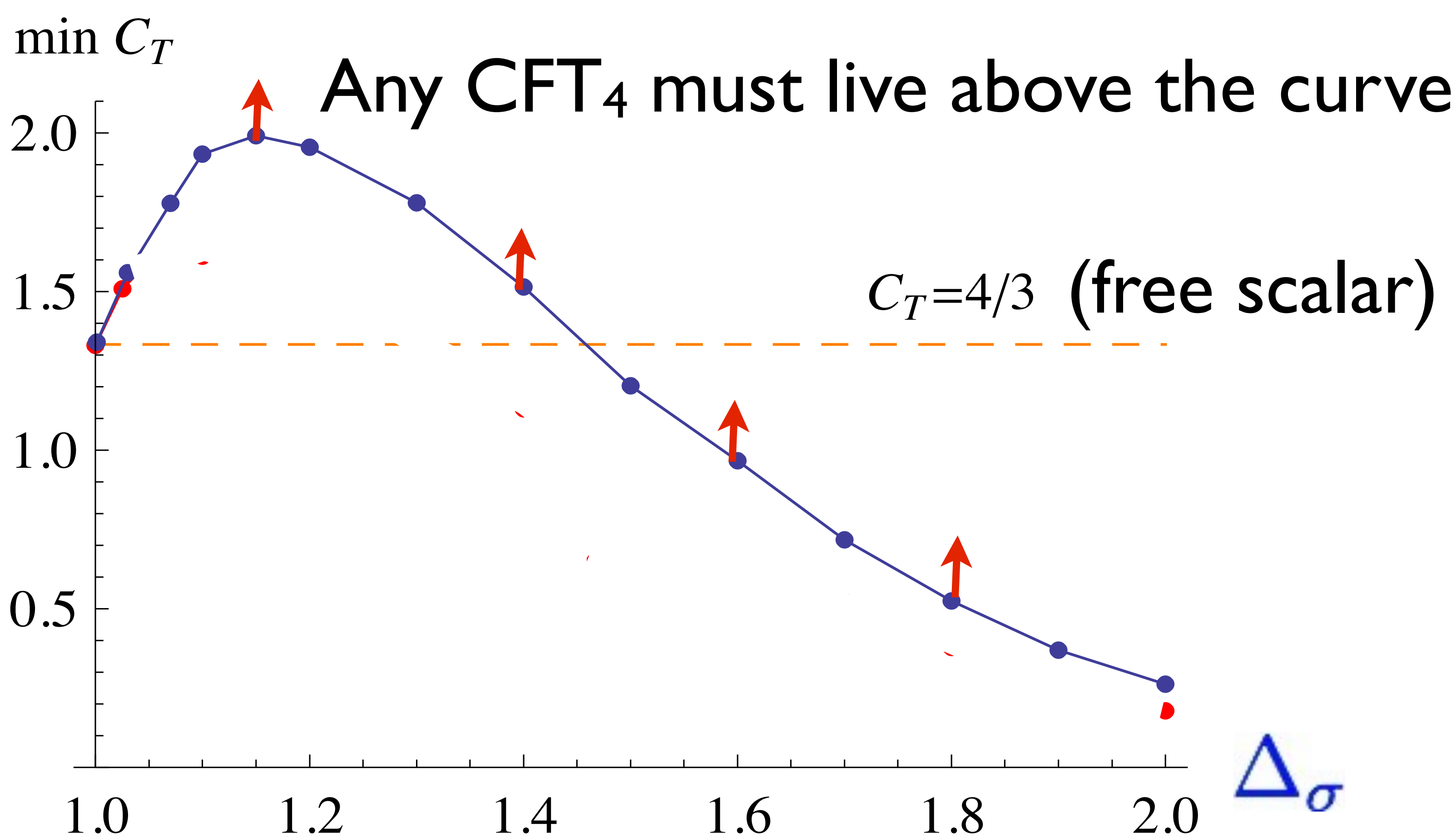
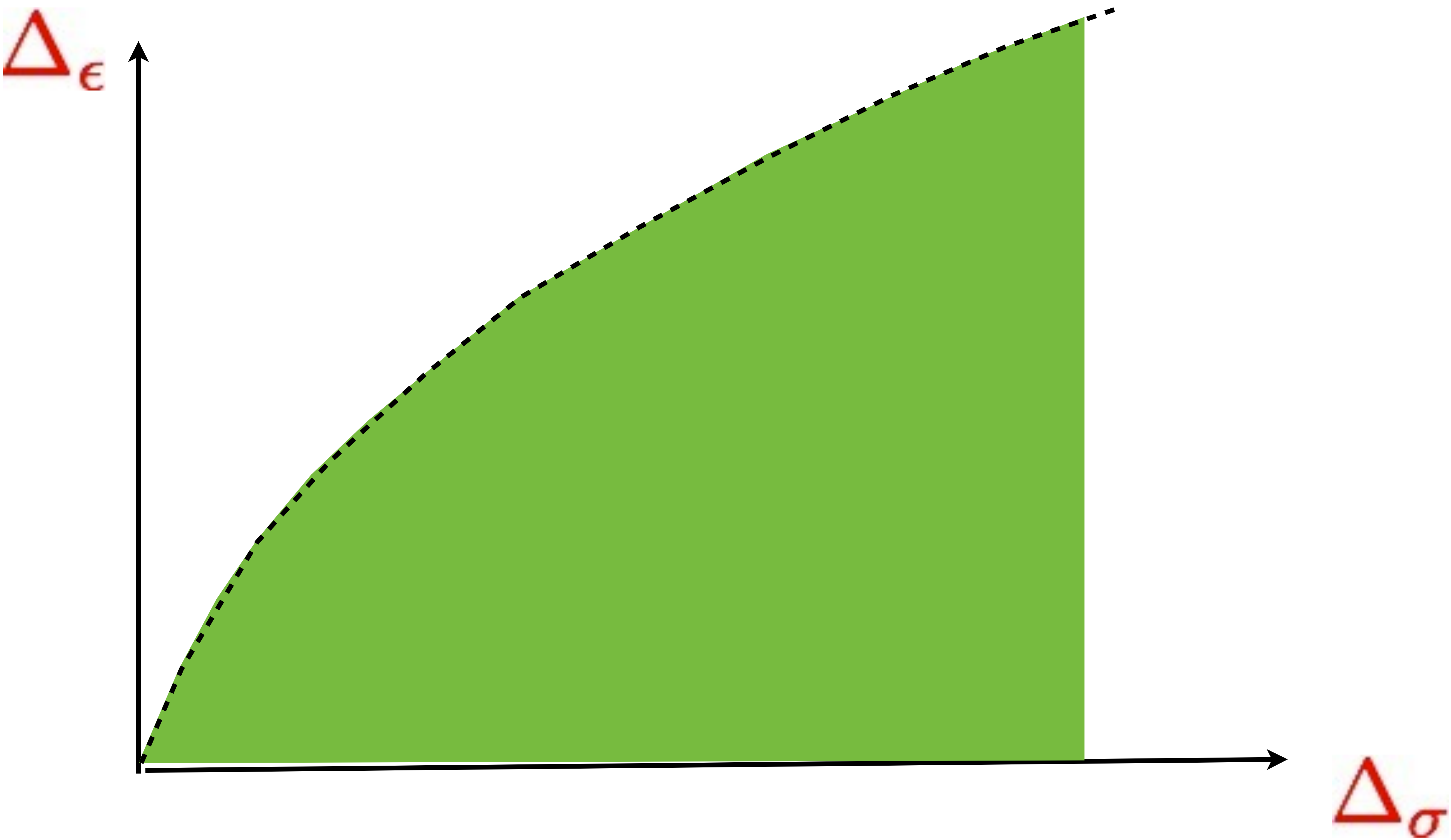


[Poland, Simmons-Duffin 2011,
Rattazzi, S.R., Vichi 2011]

II. Studying "swampland boundary"

Example: $\sigma \times \sigma = 1 + \epsilon + \ldots$



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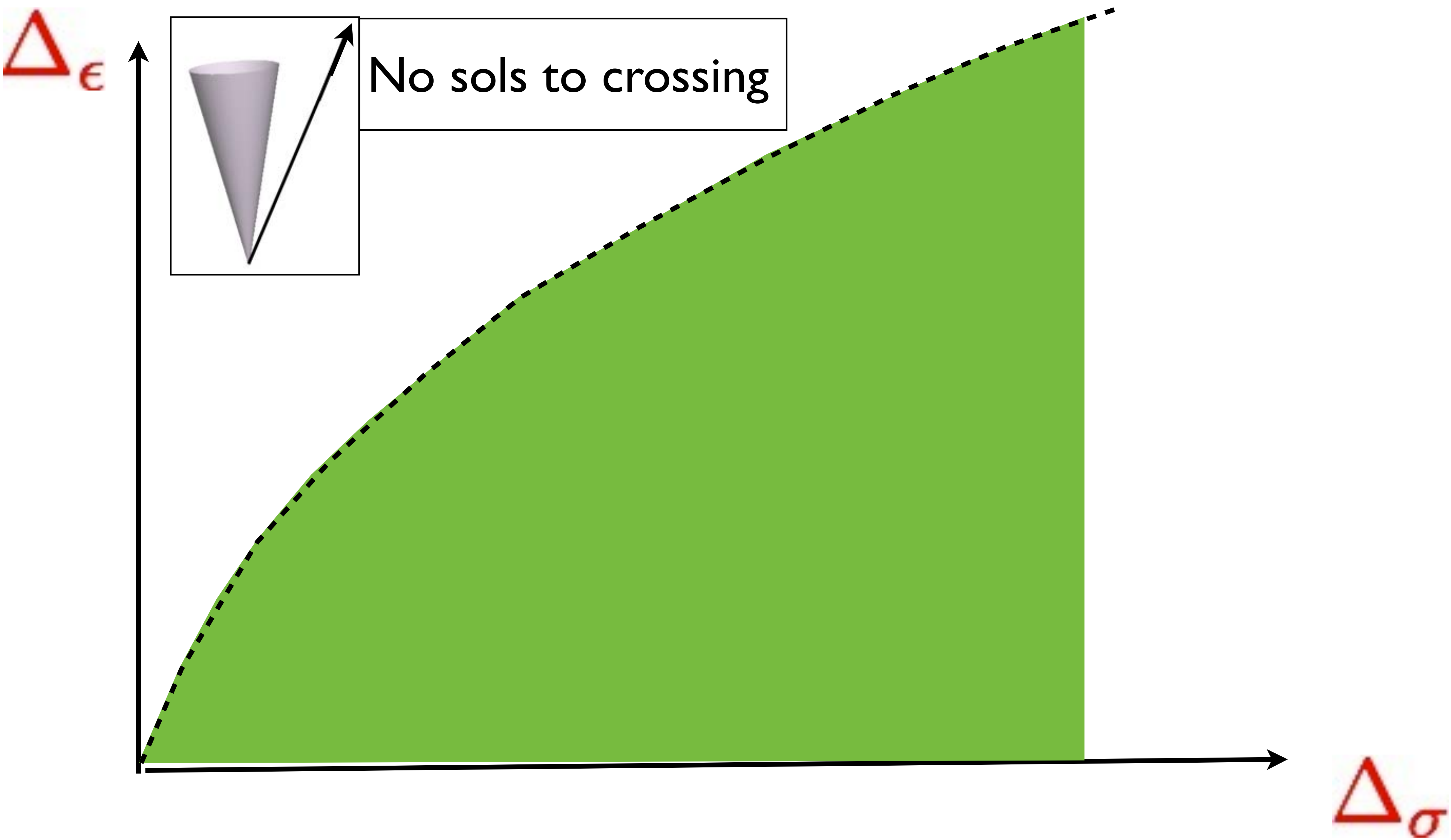

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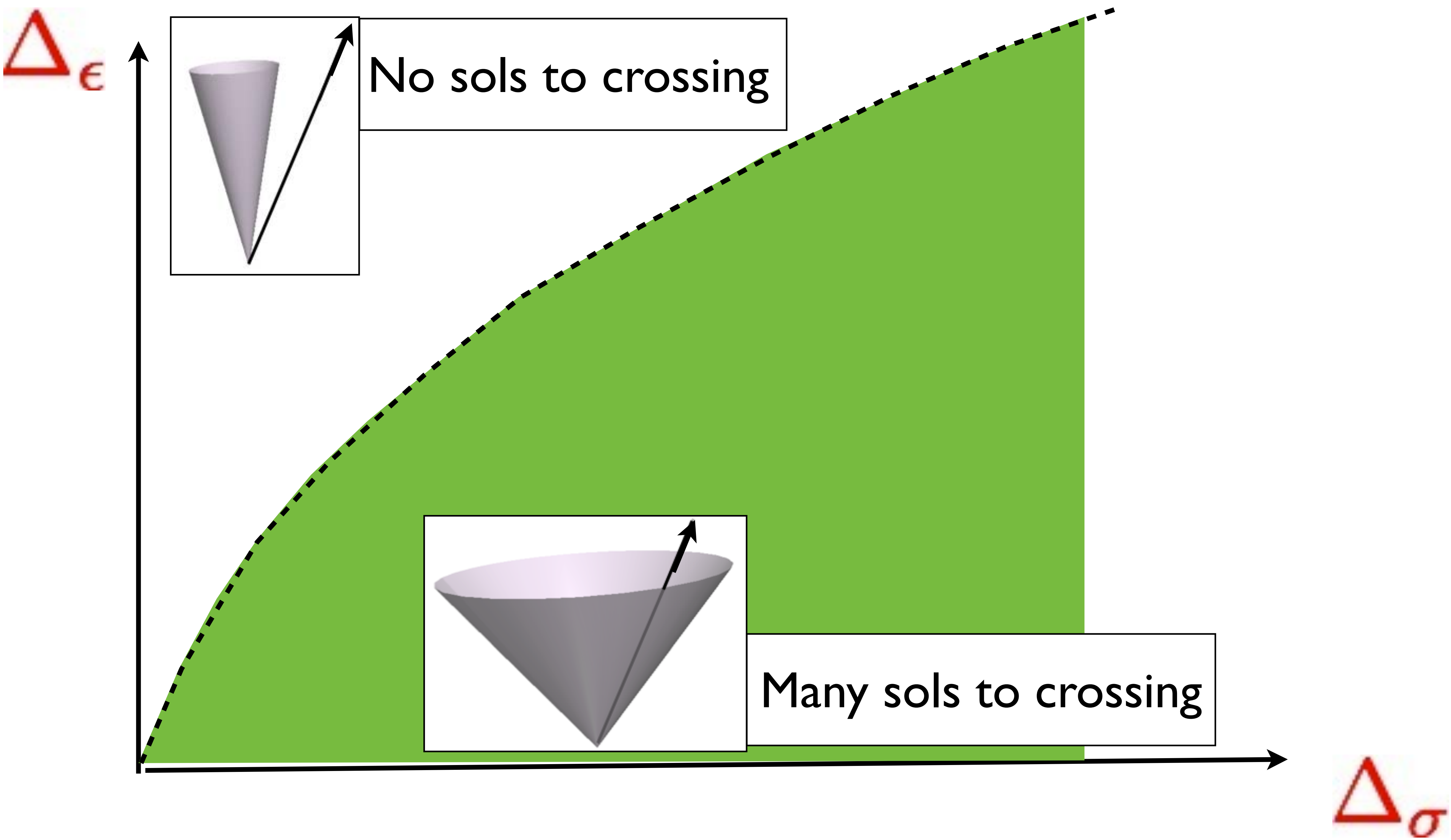

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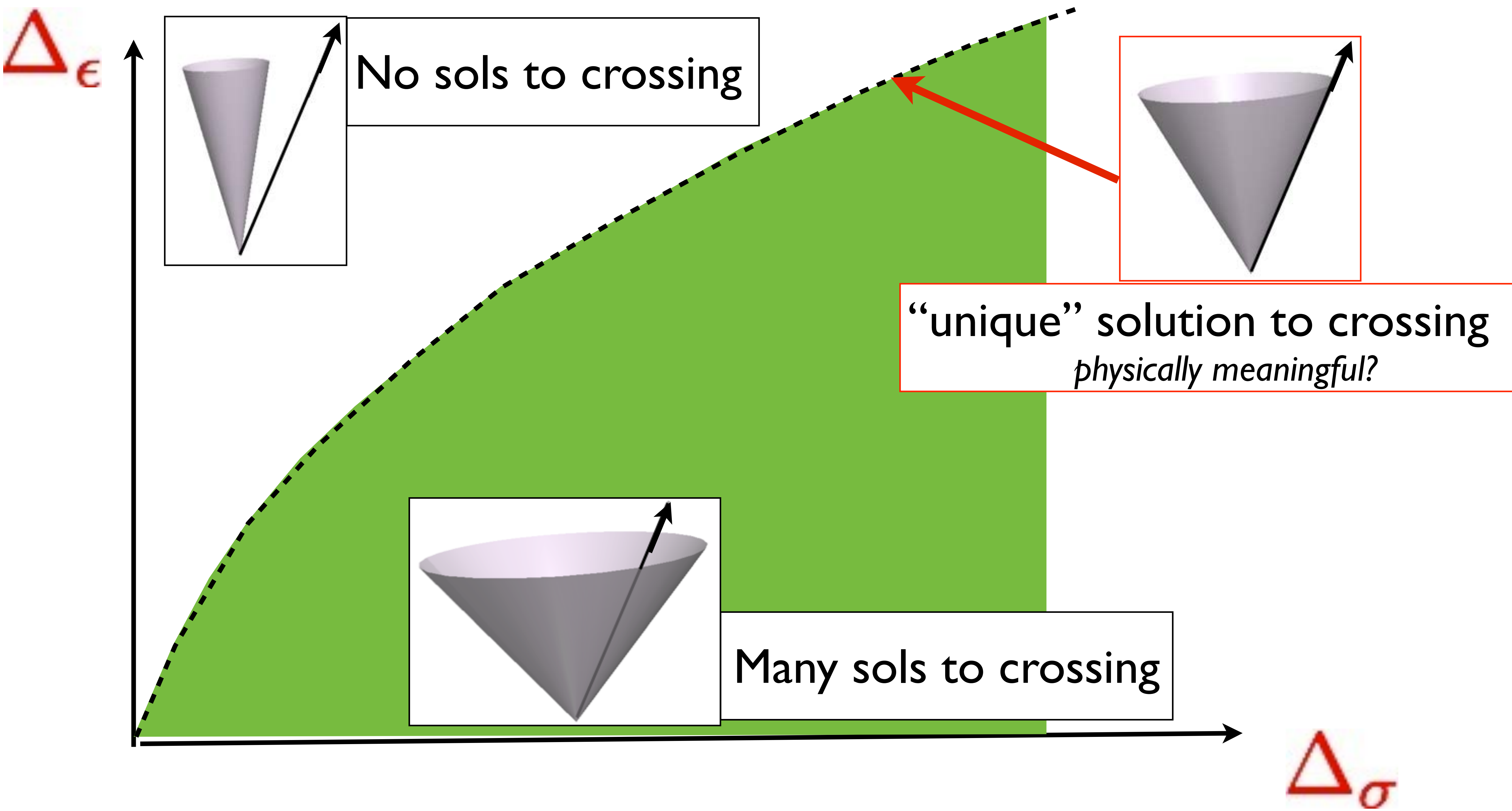
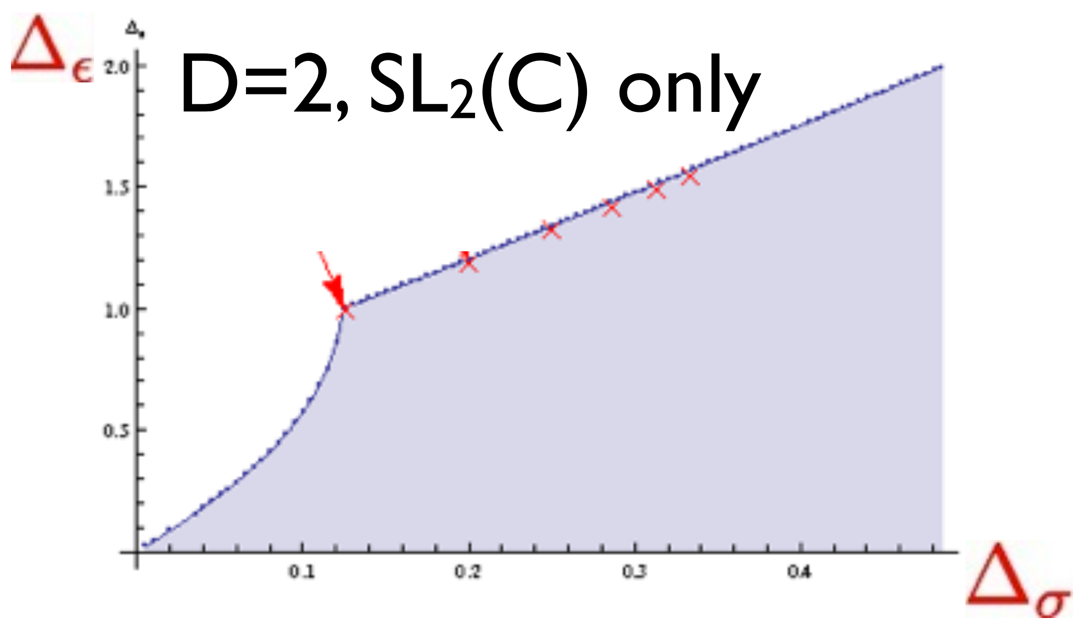
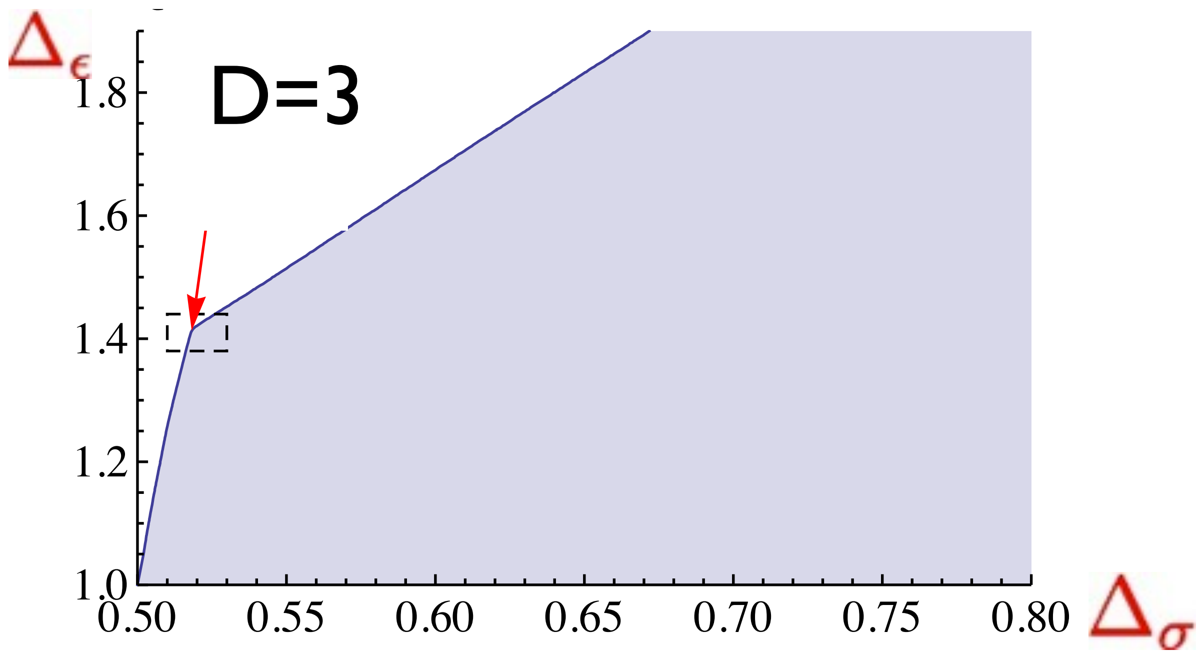


2D and 3D gap study $\sigma \times \sigma = 1 + \epsilon + \dots$

S.R., Vichi 2009; El-Showk, Paulos 2012



El-Showk, Paulos, Poland, Simmons-Duffin, S.R, Vichi'12



easy to increase precision...

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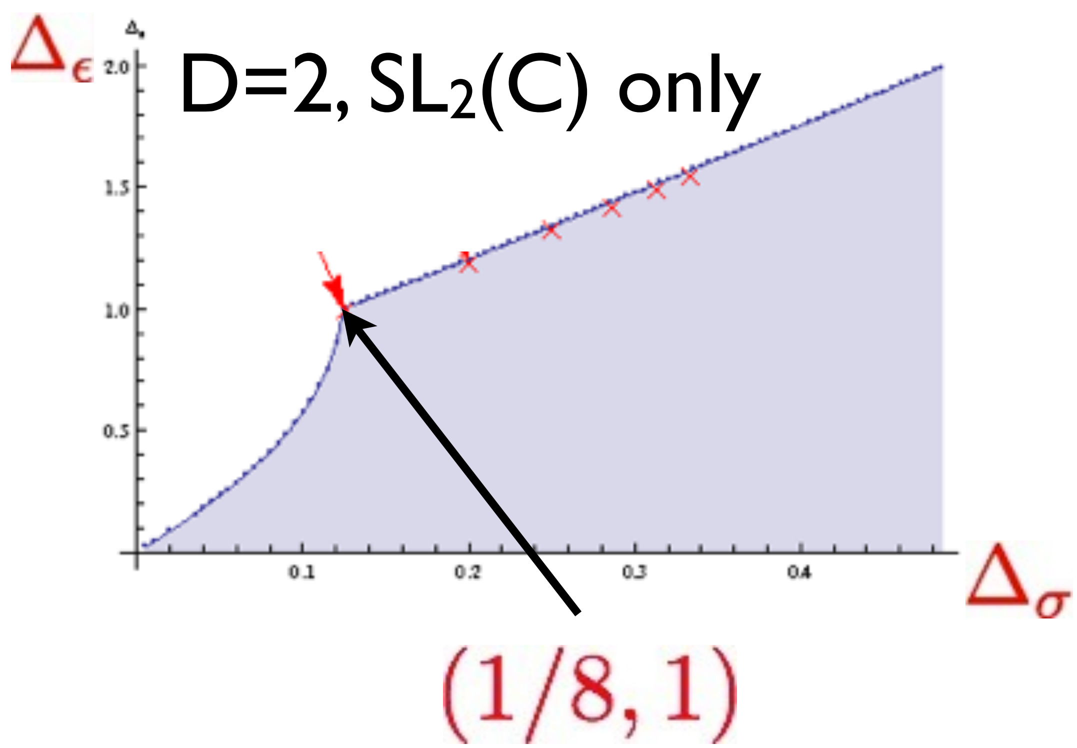

[2D Ising model]

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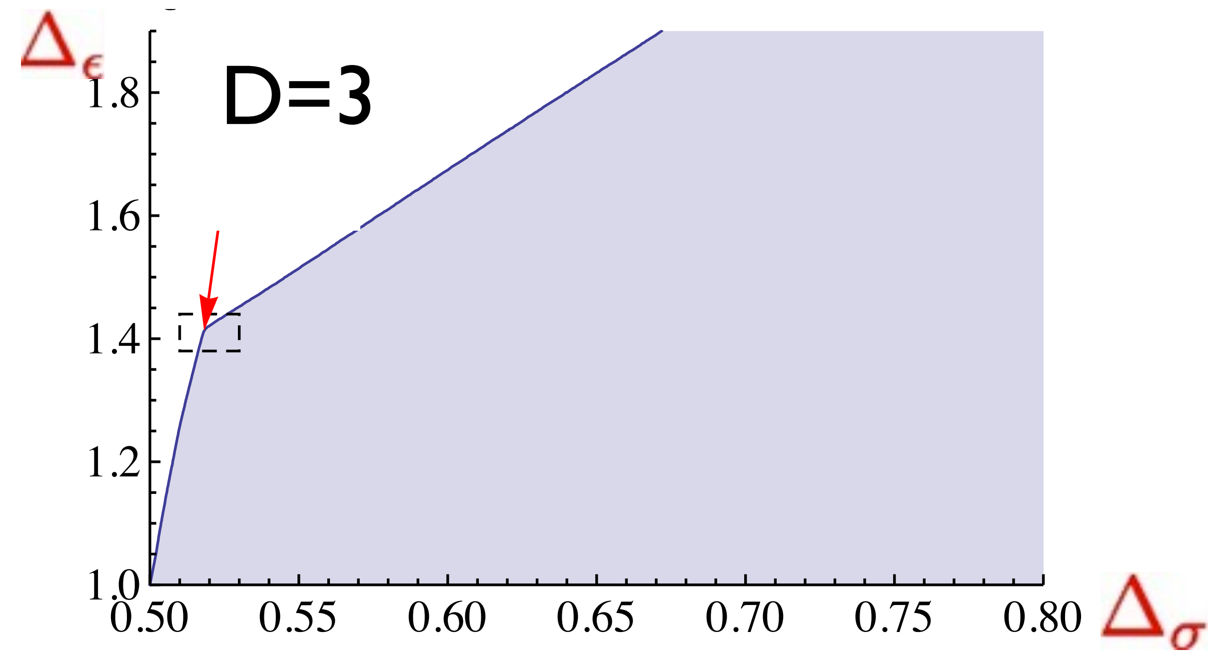

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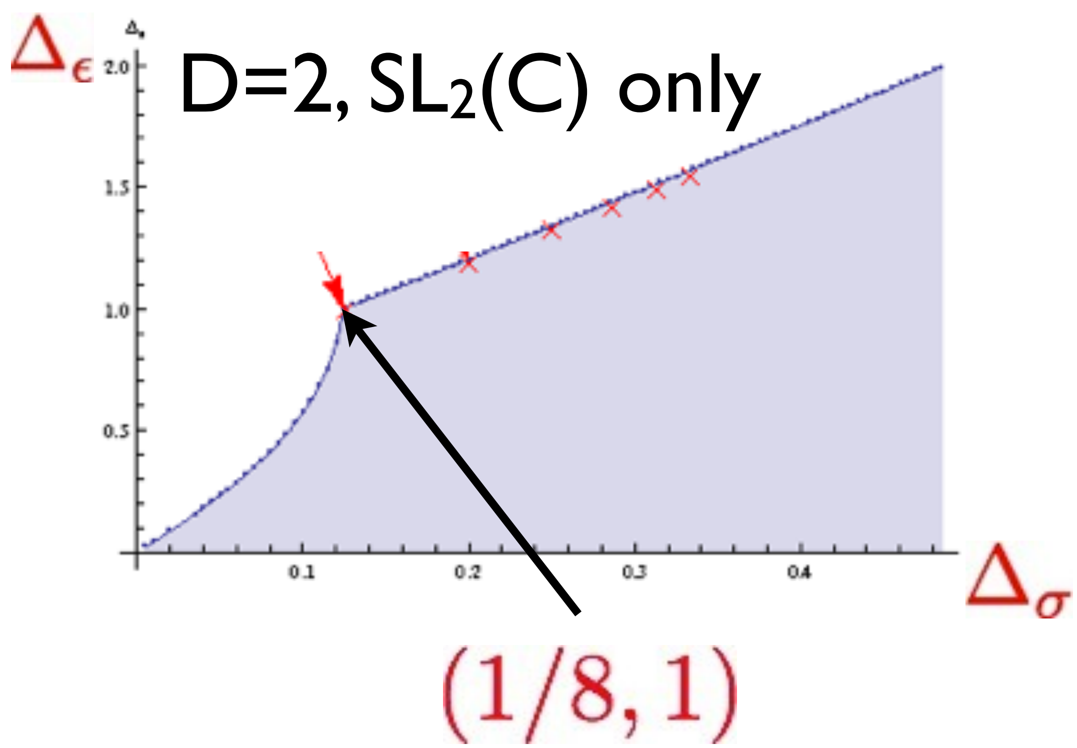

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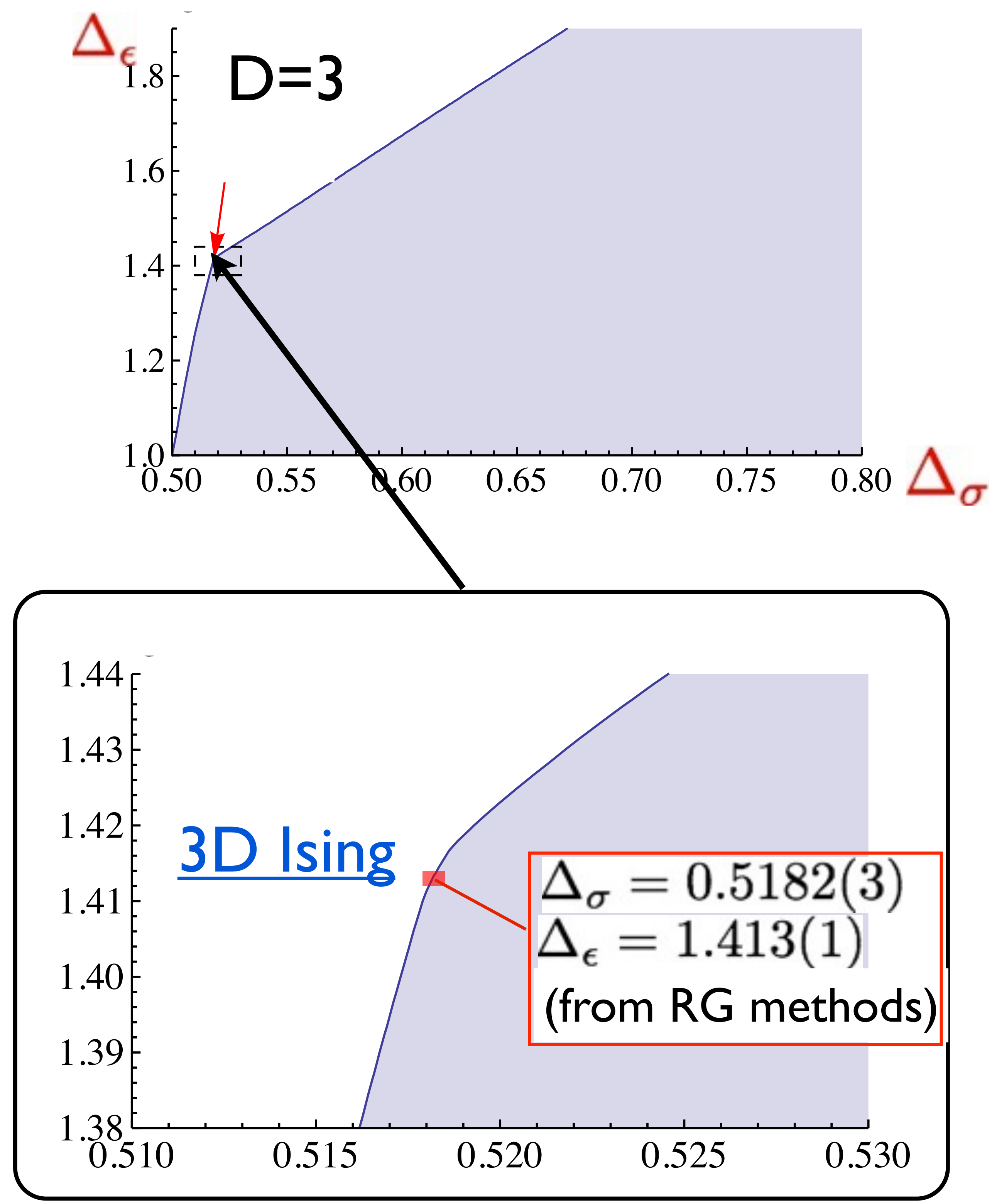

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Other kinks

- same kink happens for any $2 \leq D < 4$;
  its position agrees with ε-expansion for $D \to 4$

  [El-Showk, S.R, Vichi, work in progress]

- same kink happens for O(N) model in D=3;
  its position agrees with 1/N expansion for $N \to \infty$

  [Poland, Simmons-Duffin, work in progress]

Kinks have something to do with operator decoupling...

Spectrum of σ x σ OPE in 3D Ising model

Current knowledge (from RG methods):

Operator	Spin l	Δ
ε	0	1.413(1)
ε'	0	3.84(4)
ε''	0	4.67(11)
$T_{\mu\nu}$	2	3
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Assuming 3D Ising lives at the kink

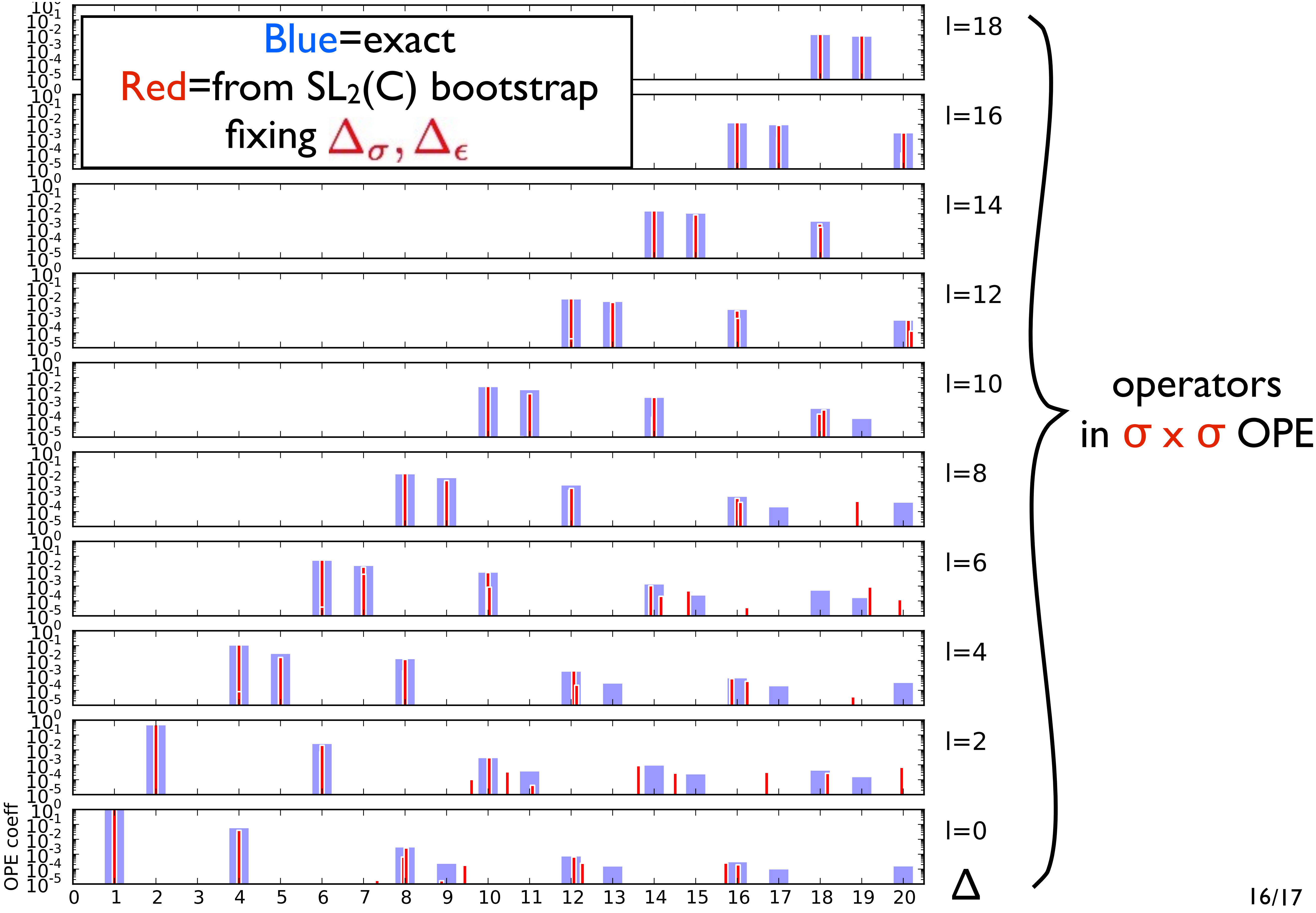
⇒ can determine all*) operators in σ x σ OPE + their OPE coeffs

[work in progress]

*) numerical work. In practice: all ≈ 20-30

Warmup study for 2D Ising

El-Showk, Paulos 2012



Other interesting developments

- Analytic results about $\ell>>1$ spectrum from bootstrap near light cone

  Fitzpatrick,Kaplan,Poland,Simmons-Duffin 2012,
  Komargodski, Zhiboedov 2012

- Bootstrap for conformal boundary conditions and defects

  [Liendo, Rastelli, van Rees 2012
  Gaiotto, Paulos, work in progress]

- Bootstrap for <JJJJ> and <TTTT> [work in progress by Dymarsky]

- Bootstrap for SUSY theories

  -N=1 Poland,Simmons-Duffin 2010 + subsequent work
  -N=4, N=2 Beem, Rastelli, van Rees 2013 + work in progress