Recent Advances in SUSY

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thanks to feedbacks from **Moore, Seiberg, Yonekura**

Sometime, a few months ago.

The Elders of the String Theory:

We would like to ask you to review the recent progress regarding "**exact results in supersymmetric gauge theories**".

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I never heard back.

So, I would split the talk into five parts, covering

$$D\text{-dimensional SUSY theories for } D = 2, 3, 4, 5, 6$$

in turn. Each will be about 10 minutes, further subdivided according to the number of supersymmetries.

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I'm joking. That would be too dull for you to listen to.

Instead, the talk is organized around three overarching themes in the last few years:

- **Localization**

- **'Non-Lagrangian' theories**

- **Mixed-dimensional systems**

Instead, the talk is organized around three overarching themes in the last few years:

- **Localization**
  Partition functions exactly computable in many cases.
  Checks of old dualities and their refinements.
  New dualities.
- **'Non-Lagrangian' theories**
  With no known Lagrangians
  or with known Lagrangians that are of not very useful
  Still we've learned a lot how to deal with them.
- **Mixed-dimensional systems**
  Compactification of 6d $\mathcal{N}=(2,0)$ theories ...
  Not just operators supported on points in a fixed theory.
  Loop operators, surface operators,...

Contents

Contents

Topological quantum field theory [Witten, 1988]

- 4d $\mathcal{N}=2$ theories have $\mathbf{SU(2)}_l \times \mathbf{SU(2)}_r \times \mathbf{SU(2)}_R$ symmetry.
- Combine $\mathbf{SU(2)}_r \times \mathbf{SU(2)}_R \to \mathbf{SU(2)}_{r'}$
- This gives covariantly constant spinors on arbitrary manifold.

Localization of gauge theory on a four-sphere and supersymmetric Wilson loops [Pestun, 2007]

- 4d $\mathcal{N}=2$ SCFTs can be put on S^4 by a conformal mapping.
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Are they very different? No.
[Festuccia,Seiberg, 2011] [Dumitrescu,Festuccia,Seiberg, 2012] ...

We can put a QFT on a curved manifold, because $T_{\mu\nu}$ knows how to couple to $g_{\mu\nu}$, i.e. non-dynamical gravity backgrounds.

A supersymmetric QFT

- has the energy-momentum $T_{\mu\nu}$, can couple to $g_{\mu\nu}$
- has the supersymmetry current $S_{\mu\alpha}$, can couple to $\psi_{\mu\alpha}$
- if it has the R-currrent J_μ^R, can couple to A_μ^R
- if it has a scalar component X_{AB}, can couple to M_{AB}

Depending on the type of the supermultiplet containing $T_{\mu\nu}$, can couple to various non-dynamical supergravity backgrounds.

[Witten 1988] used $g_{\mu\nu}$ and A_μ^R while [Pestun 2007] also used M_{AB}.

Take a QFT Q that is Poincaré invariant.

Consider a curved manifold M with isometry ξ.

Then $\langle \delta_\xi O \rangle = 0$ for any O.

Take a QFT Q that is **supersymmetric**.

Take a non-dynamical supergravity background M with **superisometry** ϵ.

Then $\langle \delta_\epsilon O \rangle = 0$ for any O.

Add to the Lagrangian a **localizing term**:

$$S \to S + t \int d^d x \delta_\epsilon O,$$

such that

$$\delta_\epsilon{}^2 O = 0, \qquad \delta_\epsilon O \simeq \sum_\psi |\delta\psi|^2.$$

Then

$$\frac{\partial}{\partial t} \log Z = \int d^d x \langle \delta_\epsilon O \rangle = 0.$$

In the large t limit, the integral localizes to the configurations

$$\delta\psi = 0$$

parameterized by some space $\mathcal{M} = \sqcup \mathcal{M}_i$. Then

$$Z = \sum_i \int_{\mathcal{M}_i} Z_{\text{classical}} Z_{\text{quadr. fluct.}}$$

This has been carried out in many cases.

- many papers on topologically twisted theories
- Ω-backgrounds on non-compact spaces such as $\mathbb{R}^d$,...
- S^2, $\mathbb{RP}^2$,...
- S^3, $S^3/\mathbb{Z}_k$, $S^2 \times S^1$,...
- S^4, $S^3 \times S^1$, $S^3/\mathbb{Z}_k \times S^1$,...
- S^5, $S^4 \times S^1$, general Sasaki-Einstein five-manifolds,...
- cases above with boundaries, codimension-2 operators, ...

Note that you need to specify the **full supergravity background**.

Only the **topological property of δ_ϵ^2 matters**: there are **uncountably-infinite choices** of values of the sugra background **with the same partition function.**

[Witten 1988][Hama,Hosomichi 2012]
[Closset,Dumitrescu,Festuccia,Komargodski 2013]

Many great developments on localization in the last couple of years.

For example,

- Connection to holography
  → [Freedman's talk], [Dabholker's talk]
- Better understanging of 2d non-abelian gauge theories
  → [Gomis's talk]
- Extremely detailed understanding of 3d theory on S^3
  → [Mariño's talk]
- and much more ...

Let me say a few words about **localization of 5d theories**.

Localization of five dimensional gauge theories

	minimal SUSY	maximal SUSY
susy literature	$\mathcal{N}=1$	$\mathcal{N}=2$
sugra literature	$\mathcal{N}=2$	$\mathcal{N}=4$

Caveat

- 5d gauge theories are all **non-renormalizable**.
- What do we mean by the localization of the path integral, then?

My excuses

- If there's a UV fixed point, we're just computing the quantity in the IR description
- If the non-renormalizable terms are all δ_ϵ-exact, they don't matter.
- Someone in the audience will think about it.

First note $\mathbf{tr}\, F \wedge F$ is a conserved current in 5d.

Minimal SUSY

$$\text{5d SCFT with } E_{N_f+1} \text{ symmetry.} \xrightarrow[m = 1/g^2]{\text{mass deform.}} \mathbf{SU}(2) \text{ with } N_f \text{ flavors } \mathbf{SO}(2N_f) \text{ symmetry.}$$

Instanton charge enhances the flavor symmetry.

Maximal SUSY

$$\text{6d } \mathcal{N}{=}(2,0) \text{ SCFT} \xrightarrow[m_{KK} = 1/g^2]{\text{put on } S^1} \text{5d max SYM}$$

Instanton charge is the KK charge.

Many nontrivial checks using **localization** and **topological vertex**.
Heavily uses the instanton counting. [Nekrasov]

$S^4 \times S^1$

[Kim,Kim,Lee] [Terashima] [Iqbal-Vafa] [Nieri,Pasquetti,Passerini]
[Bergman,Rodriguez-Gomez,Zafrir][Bao,Mitev,Pomoni,Taki,Yagi]
[Hayashi,Kim,Nishinaka][Taki][Aganagic,Haouzi,Shakirov]

S^5

[Kallen,Zabzine][Hosomichi,Seong,Terashima][Kallen,Qiu,Zabzine][Kim,Ki
[Imamura] [Lockhart,Vafa] [Kim,Kim,Kim] [Nieri,Pasquetti,Passerini]

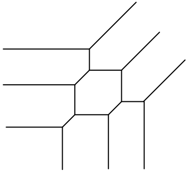
Sasaki-Einstein manifolds

[Qiu,Zabzine][Schmude][Qiu,Tizzano,Winding,Zabzine]

5d $\boldsymbol{E_6}$ theory

mass deform.

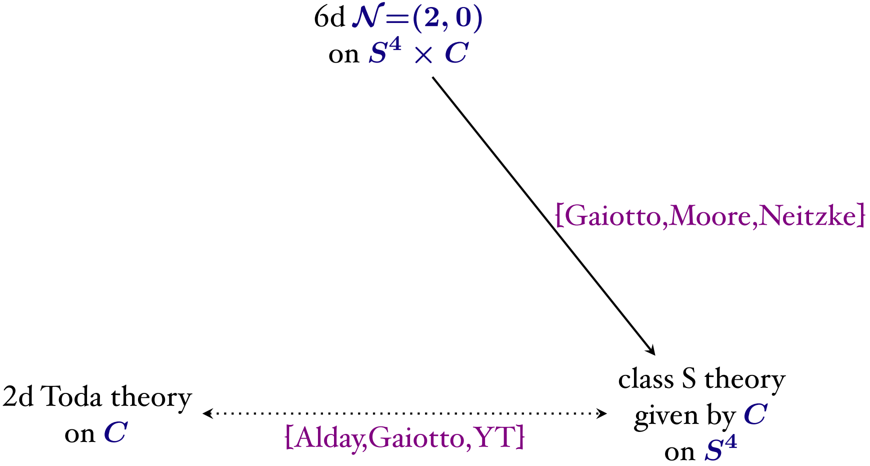
$\mathbf{SU(2)}$ with 5 flavors



$\boldsymbol{Z(S^1 \times S^4)}$ computable by gauge theory or by refined topological string

[Kim,Kim,Lee] [Bao,Mitev,Pomoni,Yagi,Taki]
[Hayashi,Kim,Nishinaka][Aganagic,Haouzi,Shakirov]

Generalization to other gauge theories
[Bergman,Rodriguez-Gomez,Zafrir]


6d $\mathcal{N}=(2,0)$
on $S^4 \times C$
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class S theory
given by C
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2d Toda theory
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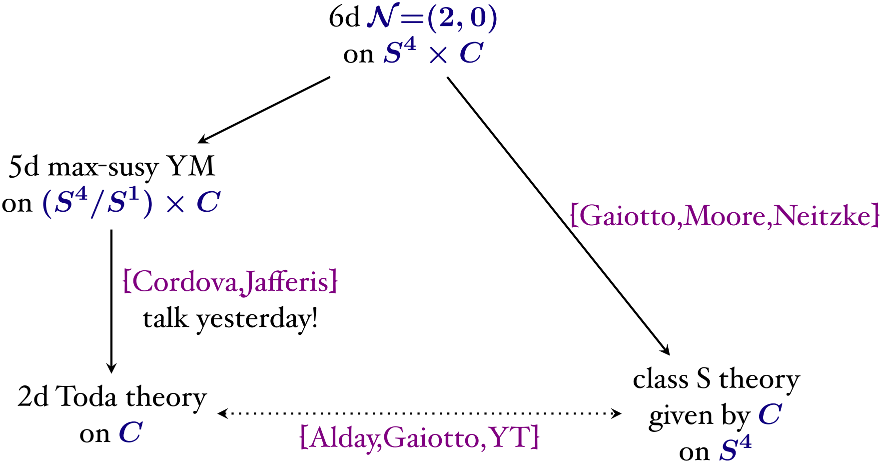
6d $\mathcal{N}=(2,0)$
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on $(S^4/S^1) \times C$
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[Cordova,Jafferis]
talk yesterday!
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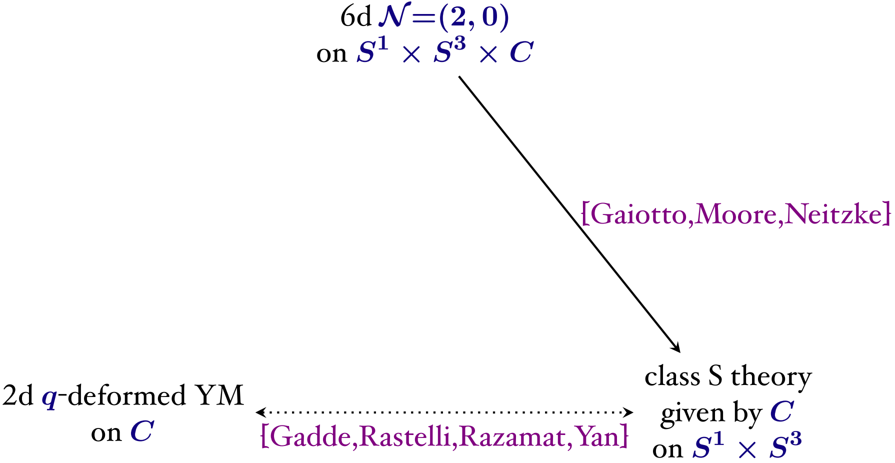
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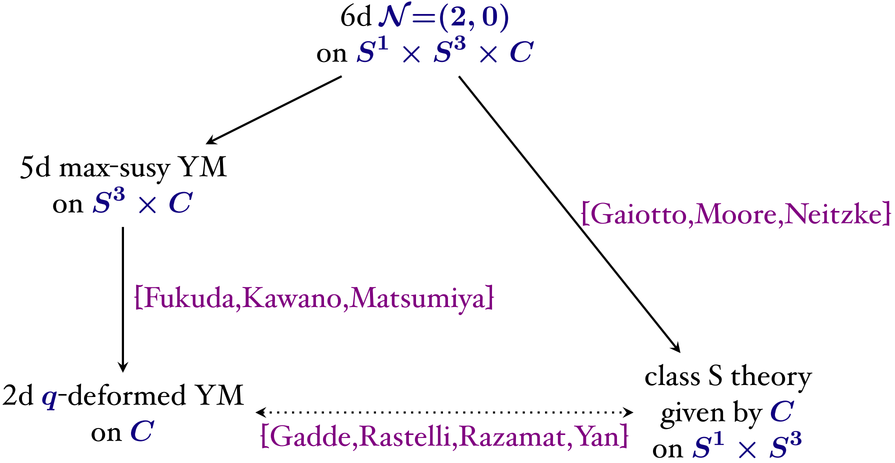
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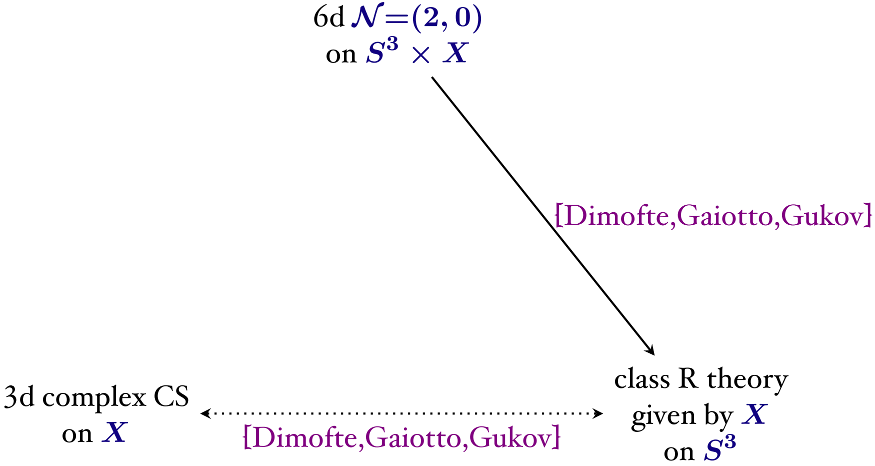
6d $\mathcal{N}=(2,0)$
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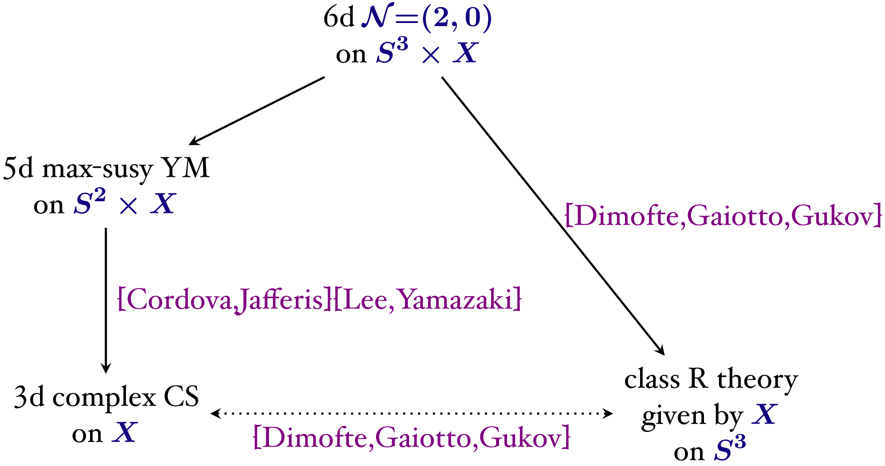
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n-dimensional susy gauge theory on $S^n \to$ matrix integral =0d QFT

n-dimensional susy gauge theory on $S^d \to (n-d)$-dimenisonal QFT

Let's call it **partial localization**.

6d $\mathcal{N}=(2,0)$ theory on $S^1 \to$ 5d max-susy YM

My gut feeling is that this is an instance of partial localization.

Contents

A **non-Lagrangian** theory, for the purpose of the present talk, is a theory such that the Lagrangian is not known and/or agreed upon.

It's a time-dependent concept.

Given a **non-Lagrangian** theory, two obvious approaches are

- to work hard to find the Lagrangian
- to work around the absence of the Lagrangian

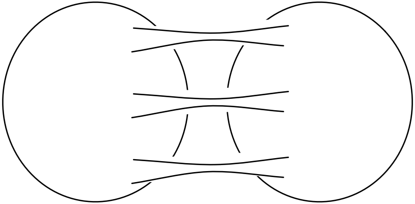
The first had a spectacular success in 3d [Schwarz,BLG, ABJM,...]

The second perspective is there for those who can't wait.

The 6d $\mathcal{N}=(2,0)$ theories are the prime examples. I'll come back to the 6d theory itself later.

First consider its compactification on a Riemann surface

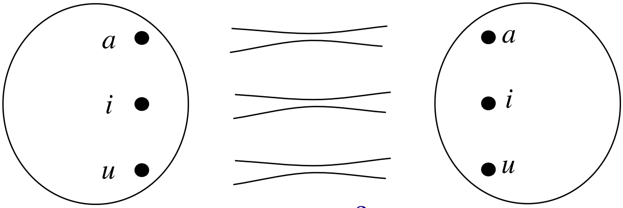
C :



and get a 4d theory. Usually non-Lagrangian.

Called the class S construction, or the tinkertoy construction. [Gaiotto,Moore,Neitzke] [Chacaltana,Distler]

Decompose it into **tubes** and **spheres** [Gaiotto]



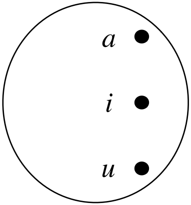
T_N $\mathbf{SU}(N)^3$ T_N

Tubes

- R-symmetry twist on C was originally chosen to preserve 4d $\mathcal{N}=2$ $\rightarrow$ $\mathcal{N}=2$ vector multiplets from tubes [Gaiotto,Moore,Neitzke][Gaiotto]
- R-symmetry twist on C can be chosen so that to have 4d $\mathcal{N}=1$ $\rightarrow$ tubes can give either $\mathcal{N}=1$ or $\mathcal{N}=2$ vector multiplets [Bah,Beem,Bobev,Wecht],[Gadde,Maruyoshi,YT,Yan],[Xie,Yonekura]

Spheres

T_N



$\mathbf{SU}(N)_1 \curvearrowright a = 1, \ldots, N$
$\mathbf{SU}(N)_2 \curvearrowright i = 1, \ldots, N$
$\mathbf{SU}(N)_3 \curvearrowright u = 1, \ldots, N$

Introduced five years ago [Gaiotto].

An 4d $\mathcal{N}=2$ theory with $\mathbf{SU}(N)^3$ symmetry.

T_2: a theory of free Q_{aiu}.

T_3: the E_6 theory of Minahan and Nemeschansky. In terms of $\mathbf{SU}(3)^3$,

$$Q_{aiu}, \tilde{Q}^{aiu}, \mu_b^a, \tilde{\mu}_j^i, \hat{\mu}_v^u, \text{ all dimension 2.}$$

T_N: not much was known.

Five years later: the spectrum of BPS operators known, thanks to the relation of the index with 2d q-deformed Yang-Mills [Gadde,Pomoni,Rastelli,Razamat,Yan].

Using that as a guide, the chiral ring relations can be worked out.

Generators on the Higgs branch side:

dimension	name
2	$\mu^a_b, \tilde{\mu}^i_j, \hat{\mu}^u_v$
$1(N-1)$	Q_{aiu}
$2(N-2)$	$Q_{[ab][ij][uv]}$
$\vdots$	$\vdots$
$k(N-k)$	$Q_{[a_1\cdots a_k][i_1\cdots i_k][u_1\cdots u_k]}$
$\vdots$	$\vdots$
$(N-1)1$	$Q_{[a_1\cdots a_{N-1}][i_1\cdots i_{N-1}][u_1\cdots u_{N-1}]} = \tilde{Q}^{aiu}$

T_N is well understood to such a degree that,
although it is **non-Lagrangian**, we can even analyze susy breaking.

- A chiral ring relation

$$\mathbf{tr}(\mu_b^a)^k = \mathbf{tr}(\tilde{\mu}_j^i)^k = \mathbf{tr}(\hat{\mu}_v^u)^k$$

  for any k.
- Couple one $\mathcal{N}=1$ $\mathbf{SU}(N)$ vector multiplet to the index a. i and u remain flavor.
- β-function = the same as $N_c = N_f$.
- Expect the deformation of the chiral ring, and indeed

$$\mathbf{tr}(\tilde{\mu}_j^i)^N = \mathbf{tr}(\hat{\mu}_v^u)^N + \Lambda^{2N}.$$

- When $N = 2$, it reproduces the deformation of the moduli space of $\mathbf{SU}(2)$ with 2 flavors.

- Add gauge singlets $\tilde{M}^i_j$ and $\hat{M}^u_v$, and add the superpotential

$$W = \tilde{M}^i_j \tilde{\mu}^j_i + \hat{M}^u_v \hat{\mu}^v_u,$$

  forcing $\tilde{\mu} = \hat{\mu} = 0$.
- This contradicts the deformation of the chiral ring

$$\mathrm{tr}(\tilde{\mu}^i_j)^N = \mathrm{tr}(\hat{\mu}^u_v)^N + \Lambda^{2N}.$$

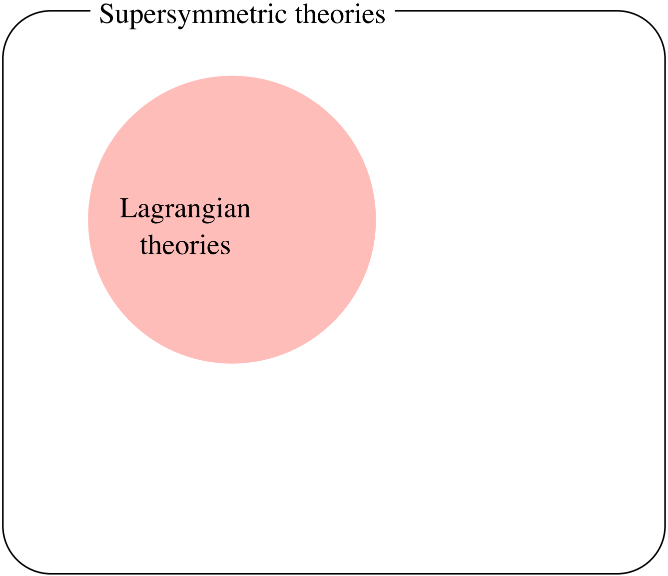
  and breaks the supersymmetry. You can check there's no run-away.
- When $N = 2$, this is the susy breaking mechanism of [ITIY]. Typically, various phenomena known to work for $\mathbf{SU}(2) = \mathbf{Sp}(1)$ and in general $\mathbf{Sp}(N)$, but not for $\mathbf{SU}(N)$, are now possible if we use T_N instead of the fundamentals.
  [Gadde,Maruyoshi,YT,Yan][Maruyoshi,YT,Yan,Yonekura]

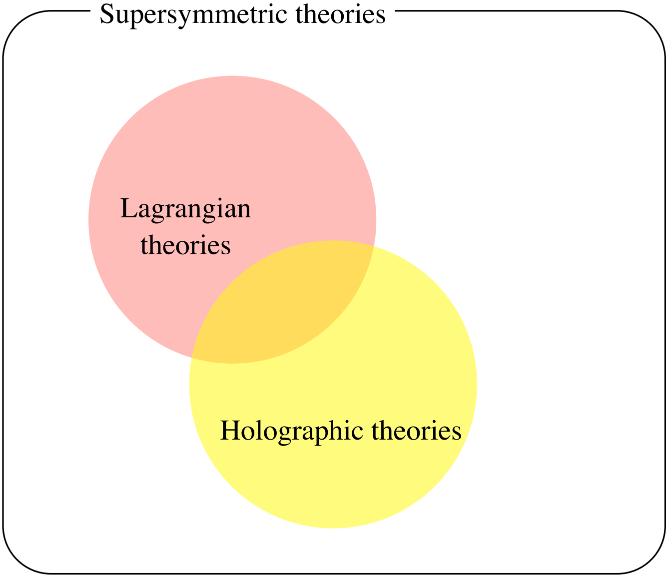
My personal impression is that by allowing T_N and other **non-Lagrangian materials**, we can have lots more fun in doing supersymmetric dynamics.

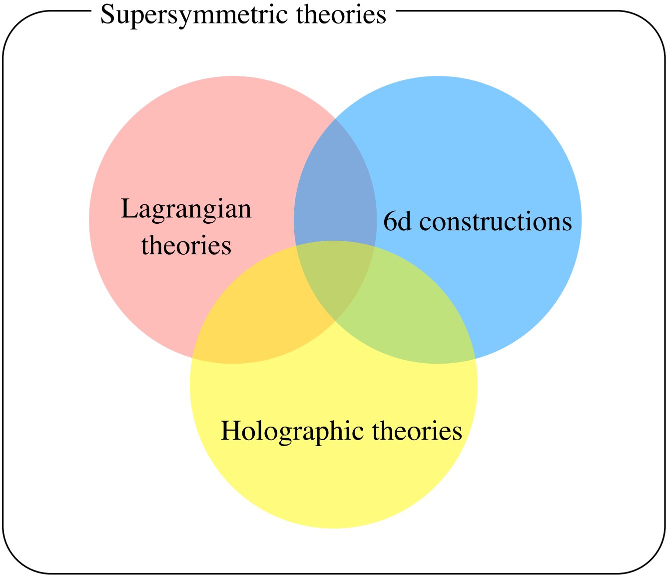
- T_N and its variants
- Generalized Argyres-Douglas theories [Zhao,Xie]
- (Γ, Γ') theories [Cecotti,Vafa,Neitzke]
- $D_p(G)$ theories [Cecotti,Del Zotto,Giacomelli]

The known ones are $\mathcal{N}=2$, but we can mix it with $\mathcal{N}=1$ gauge fields etc.

There will be genuine $\mathcal{N}=1$ non-Lagrangian materials, too.


Supersymmetric theories
Lagrangian theories


Supersymmetric theories
Lagrangian theories
Holographic theories


Supersymmetric theories
Lagrangian theories
6d constructions
Holographic theories

patching two disks

$$dr^2 + r^2 \sin^2 \theta d\theta^2$$

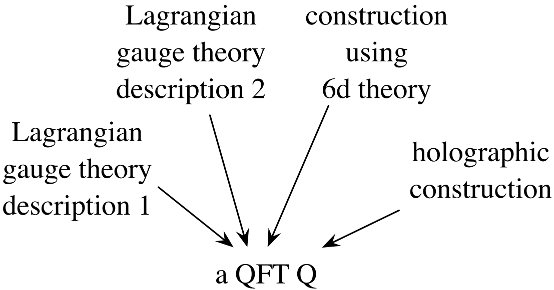
$$\{(z, w) \sim (cz, cw)\}$$

$$x^2 + y^2 + z^2 = 1$$

$$S^2$$

each can give complementary info

no one thing privileged



each can give complementary info
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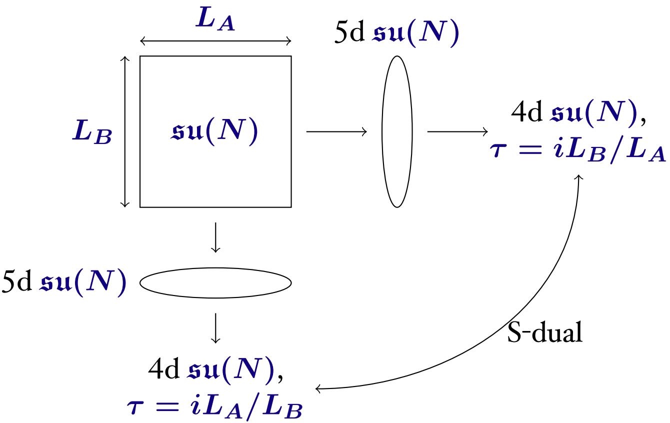
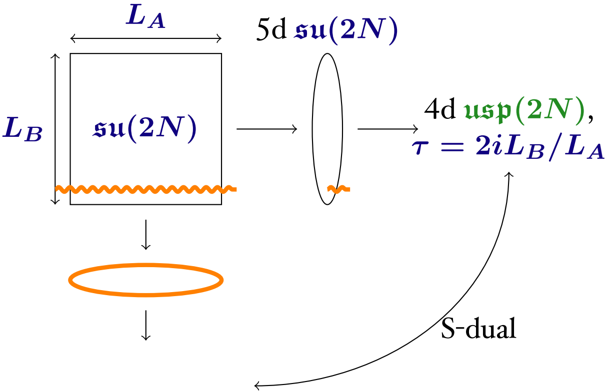
Contents

Let's now talk about the 6d theory itself. Recall the basics:



Note that $\mathfrak{su}(N)$ has $\mathbb{Z}_2$ symmetry $M \to M^T$. Using this, we find



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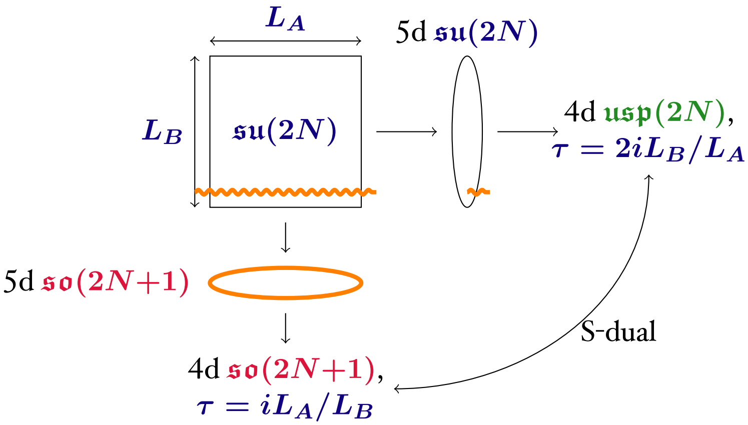

6d $\mathcal{N}=(2,0)$ theory of type $\mathfrak{su}(2N)$ has a $\mathbb{Z}_2$ symmetry, such that

6d $\mathfrak{su}(2N)$ theory

S^1 without $\mathbb{Z}_2$ twist → 5d $\mathfrak{su}(2N)$ theory

S^1 **with** $\mathbb{Z}_2$ twist → 5d $\mathfrak{so}(2N+1)$ theory

Note that $\mathfrak{so}(2N+1) \not\subset \mathfrak{su}(2N)$.

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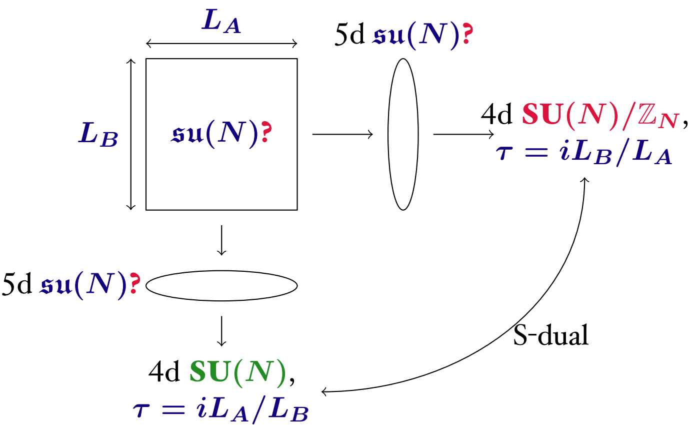
Note that $\mathfrak{so}(2N+1) \not\subset \mathfrak{su}(2N)$.

- Have you written / are you reading a paper on the Lagrangian of 6d $\mathcal{N}=(2,0)$ theory?
- If so, take 6d theory of type $\mathfrak{su}(2N)$.
- Put it on S^1 with $\mathbb{Z}_2$ twist.
- Does your Lagrangian give $\mathfrak{so}(2N+1)$?

Next, Let's study the question



6d $\mathcal{N}=(2,0)$ theory of type $\mathfrak{su}(N)$
doesn't have a unique partition function.

It only has a **partition vector.**

It's slightly outside of the concept of an ordinary QFT.

[Aharony,Witten 1998][Moore 2004][Witten 2009]

For a 4d $\mathfrak{su}(N)$ gauge theory on X, we can fix the magnetic flux

$$a \in H^2(X, \mathbb{Z}_N)$$

and consider $Z(X)_a$.

Consider 6d $\mathcal{N}=(2,0)$ theory of type $\mathfrak{su}(N)$ on a 6d manifold M.

One wants to fix

$$a \in H^3(M, \mathbb{Z}_N)$$

so that $\int_C a \in \mathbb{Z}_N$ is the magnetic flux through C.

Due to self-duality, you **can't do that** for
two intersecting cycles C, C' with $C \cap C' \neq 0$,
because they're **mutually nonlocal.**

Instead, you need to do this:

- Split $H^3(M, \mathbb{Z}_N) = A \oplus B$, so that
$$\int_M a \wedge a' = 0 \text{ for } a, a' \in A\,,$$
$$\int_M b \wedge b' = 0 \text{ for } b, b' \in B\,.$$
- Then, you can specify the flux $a \in A$ or $b \in B$, but not both at the same time.
- Correspondingly, we have
$$\{Z(M)_a | a \in A\} \text{ and } \{Z(M)^b | b \in B\}$$
related by
$$Z_a \propto \sum_b e^{i \int_M a \wedge b} Z^b.$$

This can be derived/argued in many ways.
But I don't have time to talk about it today.

In other words, there is a **partition vector** $|Z\rangle$ such that

$$Z_a = \langle Z|a\rangle, \qquad Z^b = \langle Z|b\rangle,$$

where

$$\{|a\rangle; a \in A\} \quad \text{and} \quad \{|b\rangle; b \in B\} \quad \text{with } \langle a|b\rangle = e^{i\int_M a\wedge b}$$

are two sets of basis vectors.

It's rather like conformal blocks of 2d CFTs. [Segal]

Theories that have partition vectors rather than partition functions are called under various names: **relative QFTs**, **metatheories**, etc ...

[Freed,Teleman] [Seiberg]...

6d theory of type $\mathfrak{su}(N)$ is **slightly meta**.

So, if it's just put on T^2, it's still **slightly meta**.

On $M = T^2 \times Y$, you need to write $T^2 = S^1_A \times S^1_B$, and split

$$H^3(M, \mathbb{Z}_N) \supset H^2(Y, \mathbb{Z}_N)_A \oplus H^2(Y, \mathbb{Z}_N)_B,$$

and declare you take $H^2(Y, \mathbb{Z}_N)_A$.

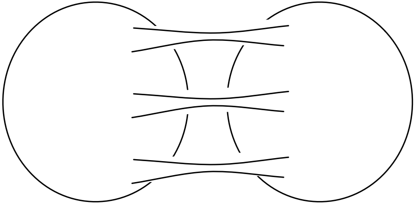
You need to make this choice
in addition to the choice of the order of the compactification.

This choice picks a particular geniune QFT, by specifing
a particular gauge group $\mathbf{SU}(N)/\mathbb{Z}_k$ and discrete θ angles
discussed in [Aharony,Seiberg,YT].

Reproduces the S-duality rule of [Vafa,Witten].

This analysis can be extended to all class S theories. [YT]

6d theory on a genus g surface C
= $2g$ copies of T_N theories coupled by $3g$ $\mathfrak{su}(N)$ multiplets.



You can work out

- possible choices of the group structure on $\mathfrak{su}(N)^{3g}$,
- together with discrete theta angles,
- how they are acted on by the S-duality ...

Let's put the 6d theory of type $\mathfrak{su}(N)$ on $M = S^3 \times S^1 \times C$.

As class S theory, the choice of the precise group of $\mathfrak{su}(N)$ vector multiplets doesn't matter, as there are no 2-cycles on $S^3 \times S^1$.

Still, we have

$$H^3(M) = H^3(S^3) \oplus H^3(S^1 \times C).$$

So, as components of the partition vector, we have

$$\{Z_a | a \in H^3(S^3) = \mathbb{Z}_N\}$$

and

$$\{Z^b | b \in H^3(S^1 \times C) = \mathbb{Z}_N\}$$

such that

$$Z_a = \sum_b e^{i2\pi ab/N} Z^b.$$

What are these additional labels a and b?

This means that 4d class S theory $T[C]$ has a $\mathbb{Z}_N$ symmetry.

$$Z_a = \mathbf{tr}_{\mathcal{H}_a}(-1)^F e^{-\beta H}.$$

is the partition function restricted to $\mathbb{Z}_N$-charge a.

Recall

$T[C]$ on $S^3 \times S^1 =$ 2d q-deformed $\mathfrak{su}(N)$ Yang-Mills on C.

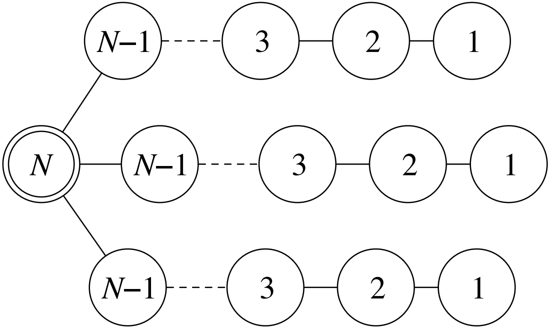
Then

$$Z^b = \sum_a e^{i2\pi ab/N} Z_a$$

is the 2d q-deformed $\mathfrak{su}(N)$ YM with **monopole flux** b on C.

The same subtlety arises in various places.

T_N on S^1 $\xleftrightarrow{\text{mirror}}$



T_N	$\leftrightarrow$	central node is $\mathbf{SU}(N)/\mathbb{Z}_N$
T_N coupled to $\mathbb{Z}_N$ gauge field	$\leftrightarrow$	central node is $\mathbf{SU}(N)$

Can be seen by performing 3d localization on S^3, $S^2 \times S^1$, lens space...
[Razamat, Willet]

These subtleties become more relevant, because with localization we can now compute more diverse quantities.

Summary

- **Localization** technique has matured.
  Gives us lots of checks of old and new dualities.

- **Non-Lagrangian theories** might have satisfactory Lagrangians in the future. But you don't have to wait.
  We are learning to analyze QFTs without Lagrangians.

- **6d $\mathcal{N}=(2,0)$ theories** are still mysterious.
  have the partition vectors, instead of the partition functions.
  Subtle but important on compact manifolds.

I would expect steady progress in the coming years.

arXiv.org > hep-th > arXiv:hep-th/9407087

Search or Arti

High Energy Physics – Theory

Monopole Condensation, And Confinement In *N*=2 Supersymmetric Yang-Mills Theory

N. Seiberg, E. Witten

(Submitted on 15 Jul 1994)

We study the vacuum structure and dyon spectrum of *N*=2 supersymmetric gauge theory in four dimensions, with gauge group SU(2). The theory turns out to have remarkably rich and physical properties which can nonetheless be described precisely; exact formulas can be obtained, for instance, for electron and dyon masses and the metric on the moduli space of vacua. The description involves a version of Olive-Montonen electric-magnetic duality. The "strongly coupled" vacuum turns out to be a weakly coupled theory of monopoles, and with a suitable perturbation confinement is described by monopole condensation.

Comments:	45pp, harvmac
Subjects:	**High Energy Physics – Theory (hep-th)**; High Energy Physics – Phenomenology (hep-ph)
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Report number:	RU-94-52, IAS-94-43

Happy 20th anniversary,

Seiberg-Witten theory!