D-brane Instantons in Supersymmetric 4D String Vacua

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(Most of the work so far was for world-sheet instantons in Type II and heterotic string theory and for M-brane instantons)

(Dine, Seiberg, Wen, Witten), (Becker^2, Strominger), (Harvey, Moore), (Witten), (Green, Gutperle), (Antoniadis, Gava, Narain, Taylor), (Rocek, Saueressig, Theis, Vandoren), (Berglund, Mayr), (Kashani-Poor, Tomasiello), (Tsimpis), (Halmagyi, Melnikov, Sethi), (Grimm) ...

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  - Stringy derivation of field theory instanton effects

Reminder: GS mechanism

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$$\prod_a U(N_a) = \prod_a SU(N_a) \times U(1)_a$$

in general contains anomalous $U(1)_a$ symmetries

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Anomaly cancellation via the 4D Green-Schwarz mechanism

- Anomalous $U(1)$s become massive and survive as global perturbative symmetries
- Only specific linear combinations of $U(1)$s are massless and remain as unbroken gauge symmetry (like $U(1)_Y$)
- Global $U(1)$ forbid some desirable matter couplings, e.g. Majorana type neutrino masses, $SU(5)$ Yukawa couplings or μ-terms $\longrightarrow$ relation to M-theory on G_2 manifolds(?)

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Consider: D2-brane (E2) instantons in Type IIA wrapping a sLag three-cycle Ξ on Calabi-Yau.

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Consider: D2-brane (E2) instantons in Type IIA wrapping a sLag three-cycle Ξ on Calabi-Yau.
From E2-E2 open strings:

- Generic 4 bosonic zero modes X_μ and 4 fermionic zero modes θ^α and $\overline{\theta}^{\dot{\alpha}}$
- Due to deformations, $b_1(\Xi)$ complex bosonic zero modes Y_i and fermionic zero modes μ_i^α and $\overline{\mu}_i^{\dot{\alpha}}$

F-terms via E2-Instantons

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F-terms possible only if

- The two $\overline{\theta}^{\dot{\alpha}}$ zero modes are projected out by $\Omega\overline{\sigma}$. For this the E2 must be invariant under $\overline{\sigma}$ and must be an $O(1)$ instanton (instead of $SP(2)$ or $U(1)$) (Argurio, Bertolini, Ferreti, Lerda, Petersson) , (Ibanez, Schellekens, Uranga) , (Bianchi, Fucito, Morales)

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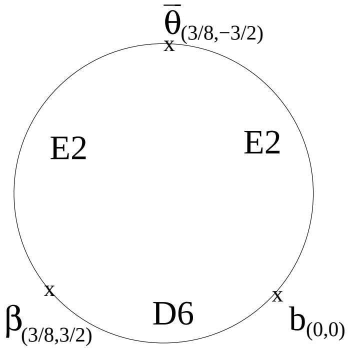
- The two $\overline{\theta}^{\dot{\alpha}}$ zero modes can be absorbed elsewhere, like for instantons on top of D6-brane:



$\longrightarrow$ fermionic ADHM-constraints (Billo et al., hep-th/0211250) ,

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Fluxes are known to lift E3-instanton zero modes (Witten), (Tripathy,Trivedi), (Bergshoeff et al.),(Lüst et al.)

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- In Type IIB $\Omega I_6(-1)^{F_L}$ orientifolds a primitive $G_{2,1}$ flux does not lift the $\overline{\theta}$ zero modes of an U(1) instanton

Type IIA Space-time Instantons

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Instanton action:

$$W_{np} \propto \quad e^{-S_{E2}} = \exp\left[-\frac{2\pi}{\ell_s^3}\left(\frac{1}{g_s}\int_{\Xi}\Re(\Omega_3) - i\int_{\Xi} C_3\right)\right]$$

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Indeed

$$e^{-S_{E2}} \longrightarrow e^{i\, Q_a(E2)\, \Lambda_a} \; e^{-S_{E2}},$$

where

$$Q_a(E2) = N_a \; \Xi \circ (\Pi_a - \Pi_a').$$

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Consequence: If $Q_a(E2) \neq 0$ for some a, no terms

$W = e^{-S_{E2}}$ possible but:

$$W = \prod_i \Phi_i \; e^{-S_{E2}} \quad \text{with} \quad \sum_i Q_a(\Phi_i) + Q_a(E2) = 0 \;\; \forall a$$

i.e. non-perturbative breakdown of global $U(1)$ symmetries.

see also e.g. : (Achucarro, Carlos, Casas, Doplicher, hep-th/0601190), (Haack, Krefl, Lüst, Van Proeyen, Zagermann, hep-th/0609211)

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How can we understand this selection rule in terms of fermionic zero modes?

Instanton zero modes

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Additional Zero modes charged under $U(1)_a$:
Strings between $E2$ and $D6_a$ have DN-boundary conditions in 4D and mixed boundary conditions along $CY_3 \longrightarrow$
1/2 complex fermionic zero mode λ_a (Ganor, hep-th/9612077)

zero modes	Reps.	number
$\lambda_{a,I}$	$(-1_E, \square_a)$	$I = 1, \ldots, [\Xi \cap \Pi_a]^+$
$\overline{\lambda}_{a,I}$	$(1_E, \overline{\square}_a)$	$I = 1, \ldots, [\Xi \cap \Pi_a]^-$
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Total $U(1)_a$ charge of all zero modes:

$$Q_a(E2) = N_a \, \Xi \circ (\Pi_a - \Pi'_a).$$

Instanton calculus

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E2-instantons are described by open strings $\longrightarrow$ computation of stringy instanton correlation functions should be possible in (boundary) conformal field theory. (Gutperle, Green, hep-th/9701093), (Billo et al., hep-th/0211250)

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As a first step we would like to compute (rigid) E2-contributions to the charged matter field superpotential

$$W_{np} \simeq \prod_{i=1}^{M} \Phi_{a_i,b_i}\, e^{-S_{E2}}.$$

with $\Phi_{a_i,b_i} = \phi_{a_i,b_i} + \theta\psi_{a_i,b_i}$ denoting chiral matter superfields at the intersection of Π_{a_i} with Π_{b_i} (suppress Chan-Paton labels for simplicity).

Instanton calculus: Summary

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Probe superpotential by correlator

$$\langle \Phi_{a_1,b_1} \cdot \ldots \cdot \Phi_{a_M,b_M} \rangle_{E2-\text{inst}} = \frac{e^{\frac{\mathcal{K}}{2}}\, Y_{\Phi_{a_1,b_1},\ldots,\Phi_{a_M,b_M}}}{\sqrt{K_{a_1,b_1} \cdot \ldots \cdot K_{a_M,b_M}}}$$

$$\boxed{\begin{aligned} &\langle \Phi_{a_1,b_1}(x_1) \cdot \ldots \cdot \Phi_{a_M,b_M}(x_M) \rangle_{E2-\text{inst}} = \\ &= \int d^4x\, d^2\theta \sum_{\text{conf.}} \prod_a \Big(\prod_{i=1}^{[\Xi\cap\Pi_a]^+} d\lambda_a^i\Big) \Big(\prod_{i=1}^{[\Xi\cap\Pi_a]^-} d\overline{\lambda}_a^i\Big) \\ &\qquad \exp(-S_{E2}) \times \exp\left(Z_0'\right) \\ &\qquad \times \langle \widehat{\Phi}_{a_1,b_1}[\vec{x}_1] \rangle^{\text{tree}}_{\lambda_{a_1},\overline{\lambda}_{b_1}} \cdot \ldots \cdot \langle \widehat{\Phi}_{a_L,b_L}[\vec{x}_L] \rangle^{\text{tree}}_{\lambda_{a_L},\overline{\lambda}_{b_L}} \times \\ &\qquad \prod_k \langle \widehat{\Phi}_{c_k,c_k}[\vec{x}_k] \rangle^{\text{loop}}_{A(E2,D6_{c_k})} \end{aligned}}$$

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Recall: loop-amplitudes uncharged (no λ_a-insertion)

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• Factor off vacuum loops involving at least one $E2$ boundary:

$$Z^A(E2, D6_a) = c \int_0^\infty \frac{dt}{t} \, \mathrm{Tr}_{E2,D6_a} \left(e^{-2\pi t L_0} \right) \neq 0$$

and likewise $Z^M(E2, O6) \neq 0$ but $Z^A(E2, E2) = 0$ (due to bose-fermi deg.).

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Therefore

$$\exp\left(Z_0\right) = \exp\left(\sum_a Z^A(E2, D6_a) + Z^M(E2, O6)\right)$$

One-loop determinants!

Instanton calculus: 1-loop

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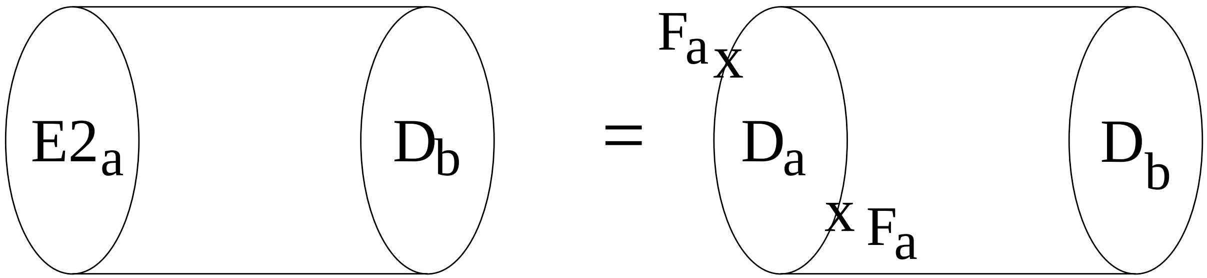
Diagrammatically we have the relation (for even spin structures)



(Abel,Goodsell), (Akerblom, BI, Lüst, Plauschinn, Schmidt-Sommerfeld)

Open problem: Computation of odd spin-structure $E2 - D6$ amplitude.

Instanton calculus: 1-loop

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Stringy one-loop amplitudes are known to include the holomorphic Wilsonian part and non-holo. contributions from wave-function normalisation

(Shifman, Vainshtein), (Kaplunovsky, Louis)

$$
\begin{aligned}
Z_0(E2_a) &= -\mathrm{Re}(f_W^a)_{1-\mathrm{loop}} - \frac{b_a}{2}\ln\left[\frac{M_p^2}{\mu^2}\right] - \frac{c_a}{2}\mathcal{K}_{\mathrm{tree}} \\
&\quad - \ln\left(\frac{V_3}{g_s}\right)_{\mathrm{tree}} + \sum_b \frac{|I_{ab}N_b|}{2}\ln\left[\det Z_{(r)}\right]_{\mathrm{tree}}
\end{aligned}
$$

with

$$
b_a = \sum_b \frac{|I_{ab}N_b|}{2} - 3, \;\; c_a = \sum_b \frac{|I_{ab}N_b|}{2} - 1.
$$

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The CFT disc amplitudes combine non-holomorphic and holomorphic pieces

$$\langle \widehat{\Phi}_{a,b}[\vec{x}] \rangle_{\lambda_a,\overline{\lambda}_b} \;=\; \frac{e^{\frac{\mathcal{K}}{2}}\, Y_{\lambda_a\, \widehat{\Phi}_{a,b}[x]\, \overline{\lambda}_b}}{\sqrt{K_{\lambda_a,a}\, \widehat{K}_{a,b}[x]\, K_{b,\overline{\lambda}_b}}} \,.$$

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Therefore, all the non-holomorphic piece including the instanton cancel out and one gets the holomorphic quantity

$$Y_{\Phi_{a_1,b_1},\ldots,\Phi_{a_M,b_M}} = \sum_{\text{conf.}} \exp(-S_{E2})_{\text{tree}} \;\; \exp\left(-f^a_W\right)_{1-\text{loop}}$$
$$Y_{\lambda_{a_1}\, \widehat{\Phi}_{a_1,b_1}[\vec{x}_1]\, \overline{\lambda}_{b_1}} \cdot \ldots \cdot Y_{\lambda_{a_1}\, \widehat{\Phi}_{a_L,b_L}[\vec{x}_L]\, \overline{\lambda}_{b_L}}\,.$$

Higher loop only contribute to corrections of Kähler potentials.

Applications : Moduli potential

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For E2-instantons with no matter field zero modes corrections to the uncharged closed/open string moduli superpotential can be generated

$$W = A(T, \Delta)\, e^{-U}$$

- Vacuum destabilisation
- KKLT like stabilisation of closed string moduli
- Inflaton potential for D-brane modulus Δ (Baumann et. al. hep-th/0607050)

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Non-pert. Majorana coupling:

$$W_{\rm M} = M_{\rm M}\,(N_R)^c\,(N_R)^c$$

with

$$M_{\rm M} = x\,M_s\,e^{-\frac{2\pi}{\ell_s^3\,g_s}{\rm Vol}_{E2}}$$

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The natural mass scale is $M_s \simeq M_{\rm GUT}$ so that $M_{\rm M}$ is non-pert. suppressed w.r.t. to $M_s >> M_{weak}$!

SU(5) Yukawa couplings

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Consider $SU(5)$ GUT model via intersecting D6-branes.

sector	number	$U(5)_a \times U(1)_b$ reps.	$U(1)_X$
(a', a)	3	$\mathbf{10}_{(2,0)}$	$\frac{1}{2}$
(a, b)	3	$\overline{\mathbf{5}}_{(-1,1)}$	$-\frac{3}{2}$
(b', b)	3	$\mathbf{1}_{(0,-2)}$	$\frac{5}{2}$
(a', b)	1	$\mathbf{5}^H_{(1,1)} + \overline{\mathbf{5}}^H_{(-1,-1)}$	$(-1) + (1)$

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Perturbative Yukawa couplings

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Perturbative Yukawa couplings

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Yukawa coupling $\langle \mathbf{10}_{(2,0)}\, \mathbf{10}_{(2,0)}\, \mathbf{5}^H_{(1,1)} \rangle$

is not $U(1)$ invariant (but present on G_2 manifolds) .

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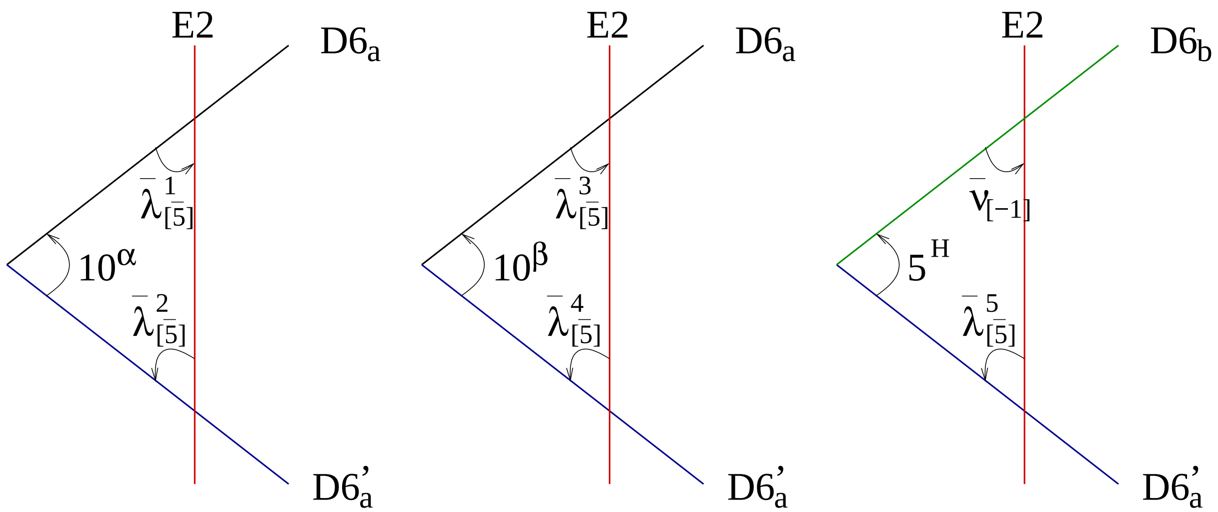
Can be generated, if the model contains an $O(1)$-instanton with $E2 \circ \pi_a = -1$ and $E2 \circ \pi_b = -1$,

(Bl, Cvetic, Lüst, Richter, Weigand, to appear)



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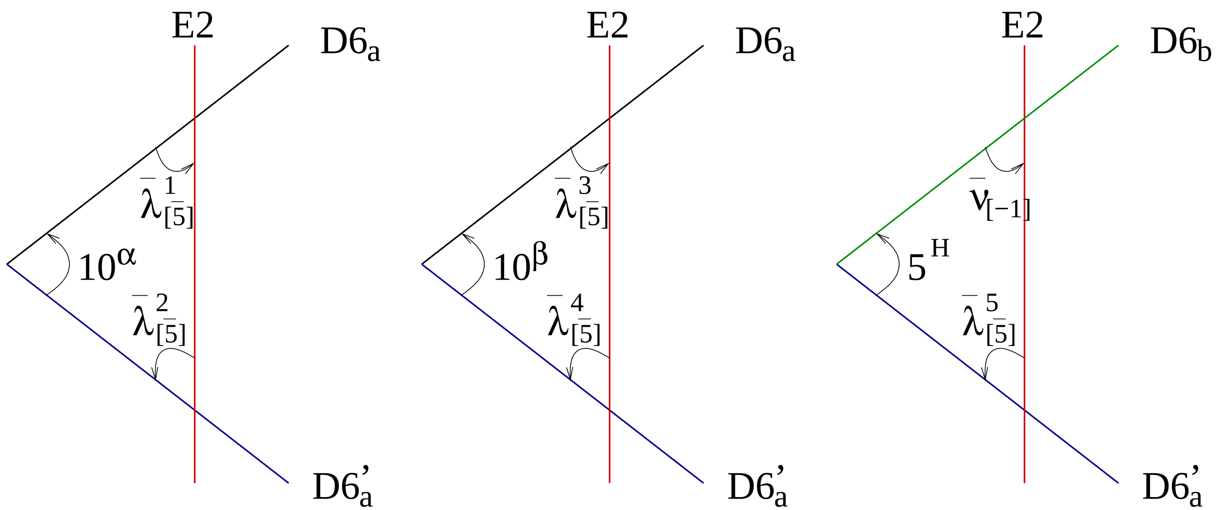

$$W_Y = Y^{\alpha\beta}_{\langle \mathbf{10}\,\mathbf{10}\,\mathbf{5}_H \rangle} \; \epsilon_{ijklm} \; \mathbf{10}^\alpha_{ij} \, \mathbf{10}^\beta_{kl} \, \mathbf{5}^H_m$$

Flipped $SU(5)$: hierarchy between (d, s, b) and (u, c, t) by E2-instanton, flavour hierarchy by world-sheet instantons

Applications: The ADS superpotential

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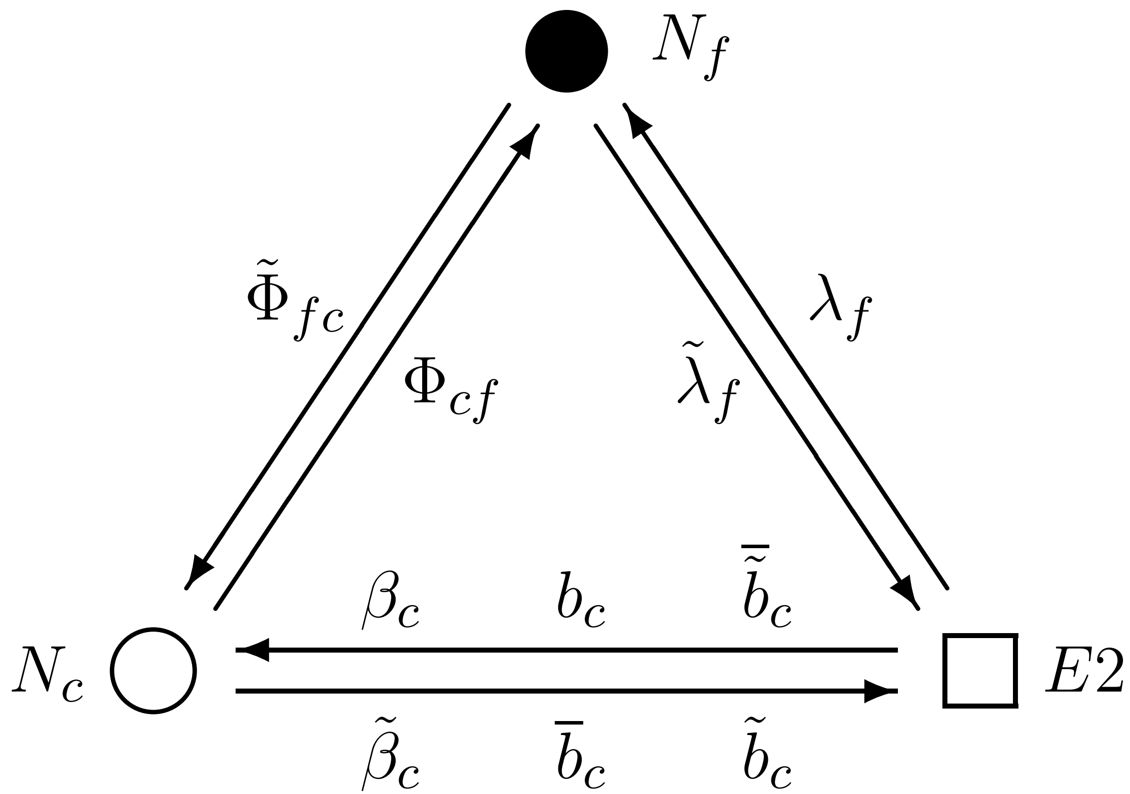
N=1 SQCD with $N_f = N_c - 1$ flavours



(Akerblom,Blumenhagen,Lüst,Plauschinn,Schmidt-Sommerfeld, hep-th/0612132)

(Florea, Kachru, McGreevy, Saulina, hep-th/0610003)

The ADS superpotential

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Issues:

- Fermionic zero modes:

$$\mathcal{L}_{\mathrm{ferm}} = \beta_c \, \overline{\Phi} \, \overline{\lambda}_f + \lambda_f \, \overline{\widetilde{\Phi}} \widetilde{\beta}_c .$$

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- ADHM constraints

Eventually one arrives at

$$S_W \simeq \int d^4x\, d^2\theta\, \frac{\Lambda^{3N_c - N_f}}{\det[M_{f f'}]}.$$

- Higher α' corrections?

generalisations (Argurio, Bertolini, Ferreti, Lerda, Petersson), (Bianchi, Fucito, Morales)

Instanton corrections to f

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Holomorphy dictates that for D6-branes the holomorphic gauge kinetic function must look like

$$f = \sum_I M_a^I U_I^c + f^{1-\text{loop}}\left(e^{-T_i^c}\right) + f^{\text{np}}\left(e^{-U_I^c}, e^{-T_i^c}\right).$$

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For intersecting D6-branes on T^6 the holomorphic one-loop gauge threshold corrections are: (Lüst, Stieberger) , (Akerblom, BI, Lüst, Schmidt-Sommerfeld)

- $\mathcal{N} = 1$ sector: $f^{(1)} = 0$
- $\mathcal{N} = 2$ sector: $f^{(1)} = \ln\left(\eta(i\,T^c)\right)$

World-sheet instanton corrections come from world-sheets with two boundaries $\rightarrow$ expect E2-instantons from non-rigid ones with $b_1(\Xi) = 1$.

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The zero mode measure reads

$$\int d^4x\, d^2\theta\, d^2y\, d^2\overline{\mu}\ e^{-S_{E2}} \ldots, \qquad \text{for } \overline{\sigma} : y \longrightarrow y$$

and

$$\int d^4x\, d^2\theta\, d^2\mu\ e^{-S_{E2}} \ldots, \qquad \text{for } \overline{\sigma} : y \longrightarrow -y.$$

(dual to world-sheet instantons studied by Beasley-Witten)

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An instanton wrapping a 3-cycle with $b_1(\Xi) = 1$ and no additional zero modes can generate a correction to the $SU(N_a)$ gauge kinetic function.

$$\begin{aligned} \langle F_a(p_1)\, F_a(p_2) \rangle_{E2} &= \int d^4x\, d^2\theta\ d^2\mu\ \exp(-S_{E2}) \\ &\quad \exp\left(Z_0'(E2)\right)\ A_{F_a^2}(E2, D6_a) \end{aligned}$$

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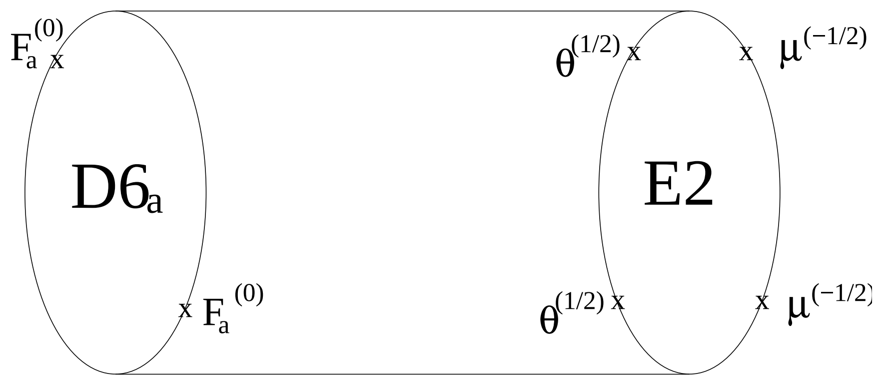
where $A_{F_a^2}(E2, D6_a)$ is the annulus diagram



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$$\xi_a = \int_{\Pi_a} \Im(\Omega_3).$$

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Expect also E2-brane instanton corrections $\rightarrow$ stability of D-branes

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