OPENING TALK

John H. Schwarz

California Institute of Technology

Strings 2013 Seoul, Korea June 24, 2013

Opening Talk

I am grateful to the organizers for giving me the opportunity of opening the Strings conference for the second year in a row. I had a great time on my one previous visit to Seoul in 1997, and I am delighted to be back.

In my opening talk last year in Munich I attempted to give an overview of the status of the field. There is little that I could add today to what I said then.

This time, instead of surveying the field, I have decided to survey a particular topic.

MULTICHARGE SUPERSTRINGS

This is a subject that I worked on in the mid-1990s and have been thinking about again recently.

My talk will be mostly a review of some superstring theory basics without a lot of esoteric mathematics. It should be relatively easy for this audience to follow. Some of you may already know all that I will say. I think this is a good way to start a conference in which many of the talks are likely to be more challenging.

Introduction and Summary

Type IIB superstring theory in 10d has two distinct string charges leading to an infinite family of half-BPS (p, q) superstrings. They form a multiplet of the $SL(2, \mathbb{Z})$ duality symmetry of the theory.

I will review basic facts about these strings as well as the nonperturbative BPS spectrum obtained after compactification on a circle. The dual description in terms of M-theory compactified on a torus will also be discussed.

Part two examines the $(2, 0)$ 6d theories associated to a set of separated parallel M5-branes. Such a theory has a finite set of BPS superstrings, which correspond to the roots of a simply-laced (ADE) Lie algebra.

In both the 10d and 6d settings there are three-string junctions that can be used to build string networks. The 6d strings (and string networks) provide an effective way of characterizing BPS brane configurations on the Coulomb branch – and also in lower dimensions after compactification on a circle or torus.

There will be additional talks about $(2,0)$ theories at this conference given by Michael Douglas and Seok Kim. Sungjay Lee might also discuss aspects of these theories in his review talk. I expect these speakers, and possibly others, to explore the subject in greater depth.

M/IIB duality

One of the dualities discovered in the 1990s relates M-theory compactified on a torus to type IIB superstring theory compactified on a circle. I will now describe how the parameters associated with each of these are related.

Let us first denote the Planck length for M-theory in 11d by l_M and the Planck length for type IIB superstring theory in 10d by l_B. The latter is related to the string scale l_S and the string coupling constant g_S by

$$l_B = g_S^{1/4} l_S.$$

The type IIB theory has a discrete $SL(2,\mathbb{Z})$ duality symmetry under which the complex scalar field

$$\lambda = C_0 + ie^{-\Phi}$$

transforms in the usual nonlinear manner. C_0 is the RR scalar and Φ is the dilaton. Let us denote the expectation value of λ by

$$\tau_B = \langle\lambda\rangle = \frac{\theta}{2\pi} + \frac{i}{g_S}.$$

The Einstein-frame metric of the type IIB theory is $SL(2,\mathbb{Z})$ invariant. Hence the Planck length l_B is also $SL(2,\mathbb{Z})$ invariant, but the string scale l_S is not.

We will characterize the M-theory torus by a modular parameter τ_M and an area $(2\pi)^2 A_M$. The type IIB circle has radius R_B.

The crucial fact about M/IIB duality is that the $SL(2,\mathbb{Z})$ duality group of the type IIB theory is identified with the modular group of the M-theory torus,

$$\tau_M = \tau_B.$$

Henceforth, both of these are referred to as

$$\tau = \tau_1 + i\tau_2.$$

Further relations are obtained by identifying dual half-BPS brane configurations and equating tensions. To do this, we need to recall that the tensions of the supersymmetric M-branes are

$$T_{M2} = \frac{2\pi}{(2\pi l_M)^3} \quad \text{and} \quad T_{M5} = \frac{2\pi}{(2\pi l_M)^6}.$$

Also, the tension of the type IIB D3-brane is given by the $SL(2,\mathbb{Z})$ invariant formula

$$T_{D3} = \frac{2\pi}{(2\pi l_S)^4 g_S} = \frac{2\pi}{(2\pi l_B)^4}.$$

Matching 2-branes and 3-branes

Equating the tension of a D3-brane with the tension of an M5-brane wrapped on the torus gives

$$l_B^4 = l_M^6 / A_M.$$

Similarly, identifying an M2-brane with a D3-brane wrapped on the circle of radius R_B gives

$$l_M^3 = l_B^4 / R_B.$$

These formulas can be combined to relate dimensionless quantities:

$$(A_M / l_M^2)^3 = (l_B / R_B)^4.$$

Matching 1-branes

The type IIB theory has an infinite spectrum of half-BPS (p, q) strings in 10d, which can be regarded as bound states of p fundamental strings and q D-strings.

The integers p and q measure the strengths with which the strings couple to the two-form fields B_2 and C_2 in the type IIB supergravity multiplet. (p, q) strings are stable whenever p and q are coprime.

The (p, q) string tensions are given by

$$T_{p,q} = |p + q\tau| T_F.$$

T_F, the tension of the fundamental string, is

$$T_F = \frac{1}{2\pi l_S^2} = \frac{1}{2\pi\sqrt{\tau_2} l_B^2}.$$

The τ dependence of these formulas is consistent with the fact that the string charges p and q transform linearly (as a doublet) under $SL(2, \mathbb{Z})$ transformations. This is required because B_2 and C_2 also transform as a doublet.

The dual description of the (p, q) strings in 9d is given by wrapping one dimension of an M2-brane on a geodesic curve in the (p, q) homology class of the torus. The length of such a cycle is

$$2\pi|p + q\tau|\sqrt{A_M/\tau_2}.$$

Multiplying this length by T_{M2} and using previous identities gives the expected result, $T_{p,q}$.

Matching 0-branes

The analysis of BPS 0-branes is more involved. Let us start with the two half-BPS cases.

An M2-brane wrapped K times on the torus is dual to K units of Kaluza-Klein (KK) excitation of the type IIB circle. This gives half-BPS states with mass

$$M_K = |K| A_M / l_M^3 = |K| / R_B.$$

The agreement of the two expressions is a consequence of previous formulas.

The little group that classifies massless excitations in D dimensions, $Spin(D-2)$, is the same as the one that classifies massive excitations in $D-1$ dimensions. Each KK excitation of a 10d massless supermultiplet on a circle is a massive supermultiplet in 9d that is identical to the massless one in 10d.

Thus, the preceding half-BPS states, which are KK excitations of the type IIB supergravity multiplet, are massive chiral supermultiplets in 9d. In particular, they contain two massive gravitinos of the same chirality.

A second class of half-BPS 0-branes arises from a (p, q) string wrapped W times on the type IIB circle. This is dual to an (n_1, n_2) KK excitation on the M-theory torus. The masses are

$$M_{n_1,n_2} = 2\pi R_B |W| T_{p,q} = \frac{|n_1 + n_2\tau|}{\sqrt{\tau_2 A_M}}.$$

The two expressions agree using previous identities for the identification

$$(n_1, n_2) = (Wp, Wq).$$

Since p and q are coprime, this means that W is the gcd of n_1 and n_2.

In this case the massive supermultiplets in 9d are identical to a massless 10d type IIA supergravity multiplet. In particular, they are nonchiral.

To summarize, we have learned that there are two distinct classes of half-BPS 0-brane massive supermultiplets in 9d: chiral ones with mass M_K and nonchiral ones with mass M_{n_1,n_2}.

Supersymmetric 0-branes in 9d that carries both types of charges can only be quarter-BPS. Let us now examine them.

A type IIB superstring on a circle with winding number W and KK excitation number K satisfies the level-matching condition

$$N_L - N_R = WK.$$

N_L and N_R denote excitation numbers of left-moving and right-moving oscillators, as usual.

String excitations with $N_L = 0$ or $N_R = 0$ are quarter-BPS. Because of the $SL(2,\mathbb{Z})$ symmetry and supersymmetry, this applies to all of the (p,q) strings.

Nine-dimensional black holes

The masses of quarter-BPS 0-branes with $WK \neq 0$ are

$$M_{K,n_1,n_2} = M_K + M_{n_1,n_2}.$$

For large $N = WK$ the number of quarter BPS states is roughly $d_N \sim \exp(2\pi\sqrt{N})$. If there is a black-hole interpretation, its entropy should be $S = 2\pi\sqrt{WK}$.

Since W is the gcd of n_1 and n_2, it jumps around wildly as the charges n_1 and n_2 are varied even though the mass does not.

This is a disturbing result, if one expects to find a macroscopic geometric description of these quarter-BPS states in terms of black holes.

The basic problem is that the charges (n_1, n_2) transform as a doublet of $SL(2, \mathbb{Z})$. The gcd W is the *only* group invariant that can be constructed out of such a doublet. Thus, any improvement of the formula, such as including contributions of multicentered configurations, would still have the same problem.

One way of avoiding this conclusion – the only one I can think of – is to suppose that these 9d quarter-BPS states do not exist. This is plausible because they are at threshold for decay into a pair of half-BPS states. The existence or nonexistence of threshold bound states is always a subtle matter. The half-BPS states do not require a macroscopic geometric interpretation.

This problem does not arise for the familiar 4d and 5d examples of supersymmetric black holes. In those cases, there are more duality group invariants, and the leading term in the entropy respects the continuous group symmetry, not just the discrete one.

String junctions and string networks

The (p, q) strings can form three-string junctions. The rules are quite simple:

- The string charges should be conserved at the junction, just like momenta at a vertex in a Feynman diagram.
- The angles between the strings are determined by requiring the forces given by the string tensions to balance at the junction.

One can use these junctions to build up string networks. The three-string junction is quarter-BPS. The same is true for any string network in a 2d plane.

By studying string networks, with the external strings ending on D3-branes, one can carry out detailed explorations of the spectrum of 4d MSYM states that preserve various amounts of supersymmetry. (4d MSYM is also known as $\mathcal{N} = 4$ super Yang–Mills theory.)

Only half-BPS states occur in the $SU(2)$ theory, since there are just two D3-branes. States with less supersymmetry occur in all of the higher-rank theories.

Last year, Sen gave a detailed discussion of quarter-BPS states in 4d MSYM theories, based on string networks. In particular, he gave a very intuitive description of wall-crossing phenomena.

Branes within branes

Sometimes we are interested in considering BPS branes that contain lower-dimensional branes embedded within them in such a way that the charge of the lower-dimensional brane is uniformly distributed within the higher-dimensional brane.

In some cases the smearing of the charge can be interpreted in terms of localized objects from a higher-dimensional viewpoint. I will give two examples.

Strings

Let us begin with the half-BPS 9d (p, q) strings. A more general possibility, which is still half BPS, is for the 10d (p, q) string to wrap the circular dimension in a helical manner.

In other words, the string has a constant winding number per unit length ρ. It is a straight line in the covering space, and therefore the string is still half BPS.

From the 9d viewpoint this produces a (p, q) string that also has finite charge densities $(\rho_1, \rho_2) = \rho(p, q)$.

By computing the 10d length of the string it is easy to deduce that the apparent tension of the string in 9d is

$$T_{p,q}(\rho) = \sqrt{1 + (2\pi R\rho)^2}\, T_{p,q} = \sqrt{T_{p,q}^2 + (\rho M_{p,q})^2},$$

where $M_{p,q} = 2\pi R T_{p,q}$, as before.

Dyonic 3-branes

Now consider an analogous construction for the D3-brane. If one of its dimensions is wrapped helically on the type IIB circle, this results in a half-BPS D3-brane in 9d with a density ρ_{M2} of M2-brane charge. The resulting tension is

$$T_{D3}(\rho_{M2}) = \sqrt{T_{D3}^2 + (\rho_{M2} T_{M2})^2}.$$

In 9d D3-brane charge and M2-brane charge are electric/magnetic duals. Thus, this D3-brane carries mutually non-local charges. This is a consequence of the self-dual charge of the D3-brane in 10d.

Part II: (2,0) Theories in Six Dimensions

It is an important challenge to develop a useful formulation of the $(2,0)$ 6d SCFTs associated with multiple coincident M5-branes. These theories, which have an ADE classification, seem to be the most fundamental of all nongravitational relativistic quantum theories.

Many interesting theories in lower dimensions, as well as relations among them, have been characterized in recent years in terms of compactifications of these theories, and this has been very fruitful.

This problem is related to the even more fundamental problem of constructing a nonperturbative formulation of M-theory/superstring theory. For example, AdS/CFT duality relates M-theory on $AdS_7 \times S^4$ with N units of four-form flux to the A_{N-1} $(2, 0)$ SCFT. This is a great correspondence, but both sides still need to be understood better.

A wide range of approaches to understanding the $(2, 0)$ SCFTs have been explored. Some of them will be discussed by other speakers.

I will describe and elaborate upon a proposal that I made in 1996, as well as work by K. Lee and H. Yee in 2006. The focus will be on identifying the BPS excitations that are present on the Coulomb branch.

A basic fact about M5-branes is that M2-branes can end on them, much like fundamental strings can end on D-branes. When an M2-brane is suspended between a pair of parallel nearby M5-branes, it can be approximated by a superstring within the 6d world-volume theory of the M5-branes. The string tension is the separation of the M5-branes times the M2-brane tension.

This is analogous to 10d superstrings suspended between a pair of parallel D3-branes. This analogy suggests that the BPS branes in the world-volume theory of a set of parallel flat M5-branes should include these strings.

Note that the 10d superstrings that enter the D3-brane analysis are transverse to the branes, whereas the 6d superstrings that enter in the M5-brane analysis are along the branes.

The (2,0) tensor multiplet

The world-volume theory of a single M5-brane has two 6d supersymmetries of the same chirality. This is analogous to the type IIB theory in 10d. In addition to 6d super-Poincaré symmetry, the M5-brane theory has a $Spin(5) = USp(4)$ R-symmetry, which corresponds to rotations of the five transverse spatial dimensions.

The superconformal symmetry that incorporates and extends these symmetries is $OSp(6,2|4)$.

The massless tensor multiplet associated with a single M5-brane consists of a 2-form field B with a self-dual field strength, four symplectic-Majorana chiral spinors, and five scalars.

In terms of the 6d massless little group, which is $SU(2) \times SU(2)$, the representations are

$$(3, 1) + 4(2, 1) + 5(1, 1).$$

The coefficients $1, 4, 5$ are the dimensions of irreducible representations of $USp(4)_R$.

Six-dimensional superstrings

Let us identify the BPS branes of a 6d $(2,0)$ theory that corresponds to N parallel M5-branes, whose 5d positions $\vec{y}_i$ are distinct but otherwise arbitrary.

There are $N(N-1)/2$ oriented half-BPS superstrings in the 6d theory. They correspond to M2-branes stretched between each pair of M5-branes (ij) with $i < j$. (Interchanging i and j reverses the orientation.) The tension of the (ij) string is proportional to the distance that the M2-brane is stretched

$$T_{ij} = T_{M2}|\vec{y}_i - \vec{y}_j|.$$

The zero modes of each superstring form a $(2,0)$ tensor supermultiplet in 6d. The supermultiplet that is the zero mode of the (ij) string contains the difference $B_i - B_j$, and the (ij) string is a source for $B_i - B_j$.

In terms of physical degrees of freedom, the tensor supermultiplet arises from tensoring left-moving and right-moving zero modes of the string as follows:

$$[(2,1)+2(1,1)]\times[(2,1)+2(1,1)] = (3,1)+4(2,1)+5(1,1).$$

This is analogous to the way that one forms the type IIB supergravity multiplet out of type IIB superstring zero modes in 10d. However, there are a few important ways in which the 6d (2,0) superstring theories differ from the 10d type IIB superstring theory:

- The 6d theories have $N(N-1)/2$ distinct oriented superstrings, whereas the IIB theory has an infinite number of distinct half-BPS superstrings – the (p,q) strings. Also, the 6d superstrings carry $N-1$ distinct charges rather than two.

More generally, the number of charges is the rank of the Lie algebra, and the charges of the strings correspond to its root vectors.

- Each B field has a self-dual field strength, which implies that each 6d superstring is self-dual. This is analogous to the D3-brane in 10d.

- The 10d fundamental string has a weak-coupling limit, which makes it possible to study the type IIB theory perturbatively. The 6d $(2,0)$ theories do not have a weak-coupling limit.

Since there is no weak coupling limit, there is not much to be gained from studying the excited modes of a free string. If one could identify circumstances in which excited string states are supersymmetric or long-lived, then this information would be useful.

Infinite straight strings, which are stable half-BPS objects, are sensible objects to consider. Since they are infinitely long, they carry infinite energy, as well as a conserved string charge. When these strings wrap compact dimensions, their energy is finite, and more of their properties become relevant.

Dual type IIB interpretation

The "geometric engineering" approach gives an alternative description of the 6d $(2,0)$ theories in a type IIB setting. Suppose that four dimensions of type IIB superstring theory form an ALE space, which is a noncompact hyper-Kahler manifold Y. For example, the A_{N-1} case is given by the hyper-Kahler resolution of $\mathbb{C}^2/\mathbb{Z}_N$.

The corresponding 6d $(2,0)$ theory decouples from the rest of the type IIB dynamics because the ALE space is noncompact.

Such a space Y contains a collection of 2-cycles, which correspond to the positive roots of an ADE Lie algebra. When the 2-cycles collapse to a point they give a singularity of ADE type. On the Coulomb branch all the 2-cycles have finite size.

Since A_{N-1} has $N(N-1)/2$ positive roots, this is the number of 2-cycles. D3-branes wrapping these 2-cycles are the 6d superstrings we have discussed. $N-1$ of these 2-cycles correspond to the simple roots, whose intersection numbers are encoded in the Dynkin diagram.

The decoupling of the center-of-mass tensor multiplet in the M-theory analysis is not an issue in this approach, because it is not present in the first place.

A related problem, in which gravity is not decoupled, is K3 compactification of the type IIB theory. In that setting there is an $SO(21, 5; \mathbb{Z})$ duality group, and strings can carry $21 + 5 = 26$ different charges. The duality group relates strings that arise from D3-branes wrapping 2-cycles to the unwrapped (p, q) superstrings. This suggests that their properties are closely related.

$SU(2)$ MSYM theory in 5d

Let us consider compactification of the A_1 $(2,0)$ theory on a circle of radius R. This is expected to give 5d $SU(2)$ MSYM with coupling constant $g_{YM}^2 = 8\pi^2 R$.

The 5d MSYM theory has recently been shown to be UV divergent at six loops (Bern et al., 2012). In principle, the $(2,0)$ construction provides its UV completion.

Let us examine whether we can account for all of the BPS particles and strings that are expected in the 5d MSYM gauge theory.

Massless particles/fields arise as the dimensional reduction of the massless tensor supermultiplet in 6d. This gives a massless vector supermultiplet in 5d.

Supersymmetric massive states can arise either as string winding modes or as KK excitations. These are both half-BPS. Just as in 9d, there are two different half-BPS massive supermultiplets in 5d.

The massive little group in 5d is the same as the massless one in 6d, namely $SU(2) \times SU(2)$.

The KK excitation massive supermultiplets are identical to the 6d massless tensor multiplet, namely

$$(3,1) + 4(2,1) + 5(1,1).$$

The winding supermultiplets are non-chiral. Just as in 10d, the relative chirality of the left and right movers is reversed for winding states yielding the content

$$[(2,1) + 2(1,1)] \times [(1,2) + 2(1,1)]$$
$$= (2,2) + 2(2,1) + 2(1,2) + 4(1,1),$$

which happens to be the same as a massless vector supermultiplet in 6d MSYM.

The winding number W is a gauge charge, since it couples to the massless vector field. The choices $W = \pm 1$ give the charged members of the broken $SU(2)$ gauge multiplet. They are often referred to as W particles, which is a fortuitous name.

The mass of a winding state is $2\pi R|W|T$. Winding states with $|W| > 1$ would give multiply-charged particles, which we certainly do not want. They are at threshold for decay into singly-charged particles. Therefore only the $W = \pm 1$ winding states should exist.

Suppose we compactify on a torus instead, to make contact with 4d MSYM. Then winding the string on a (p, q) cycle of the torus gives a half-BPS dyon, with electric charge p, magnetic charge q, and mass

$$M_{p,q} = 2\pi|p + q\tau|\sqrt{A_M/\tau_2}\,T.$$

This is the same formula we encountered previously for the tension of a (p, q) string in 9d interpreted as a wrapped M2-brane. As in the that case, p and q are co-prime, and transform as doublet of the $SL(2, \mathbb{Z})$ duality group. In a dual interpretation the 4d dyons arise from (p, q) strings stretched between two D3-branes.

Returning to the 5d analysis, the KK charge K is associated to a global $U(1)$ symmetry, since this is not a gravity theory. The masses of the KK states are K/R. They correspond to "instanton particles" in the 5d gauge theory.

Classically, the 5d gauge theory (in the unbroken phase) has standard K-instanton solutions that utilize the four spatial dimensions. Roughly, the KK states correspond to normalizable harmonic forms on the instanton moduli spaces. In a type IIA interpretation they are D0-branes inside D4-branes.

An obvious question is whether there are quarter-BPS 0-branes that carry both W charge and K charge. These would satisfy a formula of the type $N = WK$. In the 9d case I argued (based on BH considerations) that they should not exist. That argument does not apply here.

If these states do exist in 5d, then the details of the spectrum of string modes would become relevant. Kim et al. (arXiv:1110.2175) computed an index in 5d MSYM that encodes properties of such states. They found a rich spectrum, implying that these states do exist, but so far there is no simple string interpretation of their results.

Finally, there is the original 6d superstring itself, now in the 5d theory. It couples magnetically to the massless vector field. This string is described in the gauge theory on the Coulomb branch by the 't Hooft– Polyakov monopole construction. It extends along the spatial direction that is not utilized in the construction.

The string can also be wound helically along the circle. In this case, it carries an electric charge density ρ per unit length, which makes it dyonic with 5d tension $T(\rho) = \sqrt{1 + (2\pi R\rho)^2}\, T$. In 2006 Lee and Yee generalized the monopole solution of the gauge theory to the dyonic string in 5d.

String junctions and string webs

Lee and Yee also observed that the 6d superstrings can form three-string junctions. The strings at a junction are the strings associated with the three pairings of any three of the M5-branes. Moreover, in 5d there can be three-string junctions of dyonic strings.

String junctions span a plane (if the three M5-branes are not collinear), and the configuration is quarter BPS. The junctions can be combined to build up complex string networks, both planar and nonplanar.

In the A_{N-1} case the number of distinct string junctions is

$$2 \times N(N-1)(N-2)/6.$$

The factor of two accounts for orientation reversal.

One of the challenges is to account for the *number of degrees of freedom* predicted by the dual AdS construction. In the case of the A_{N-1} $(2,0)$ theories, the AdS prediction at leading order in N is $N^3/3$.

In 2011 Bolognesi and Lee observed that this corresponds to the number of three-string junctions. In fact, they also pointed out that if one adds $N(N-1)$ as the contribution of individual strings, then subleading powers of N also agree.

There is an anomaly-based derivation of the number of degrees of freedom that I find more compelling. However, if it is correct, the explanation in terms of strings and string junctions has the virtue of identifying the degrees of freedom that are being counted. It is tempting to speculate that there may be an associated algebraic structure.

Conclusion

The duality between type IIB superstring theory compactified on a circle and M-theory compactified on a torus has interesting implications for both theories, and it provides useful tools for studying theories associated with multiple parallel branes.

Even though the $(2,0)$ SCFTs in 6d interact strongly, one can identify their BPS branes on the Coulomb branch. These consist of massless tensor multiplets and half-BPS strings, as well as string junctions and networks.

The conformal limit, in which the strings become tensionless, still needs to be understood better.

I am looking forward to an exciting conference.

Thank you for your attention.