PP-STRING INTERACTIONS FROM YANG MILLS THEORY

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BASED ON WORK DONE IN COLLABORATION WITH

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OUTLINE

① LIHGTONING REVIEW OF BMN

② NONPLANAR GRAPHS SURVIVE THE BMN $N \to \infty$ LIMIT
[RENORMALIZED GENUS COUNTING PAR = $\frac{J^4}{N^2}$]

③ 3 STRING INTERACTIONS FROM YANG MILLS 3-POINT FUNCTIONS
[AT LARGE μ]

④ ONE LOOP MASS RENORMALIZATION OF EXCITED STRING STATES USING 2ND ORDER PERTURBATION THEORY

⑤ 'ANOMALOUS DIMENSIONS' OF BMN OPERATORS AND EXACT AGREEMENT WITH ④.

⑥ MIXING OF BMN OPERATORS WITH DOUBLE TRACE OPERATORS AND MODIFIED SCALING DIMENSION.
[INTERPRETATION: CONTACT TERM?]

I LIGHTENING REVIEW OF BMN.

IIB THEORY ON

$$ds^2 = -4\,dx^+dx^- - \mu^2 z^2 dx^{+2} + d\vec{z}^{\,2}$$

$$(\vec{z} = z_i \ , \ i = 1..8)$$

$$F_{+1234} = F_{+5678} = \frac{\mu}{4\pi^3 g_s \alpha'^2} \ ; \ e^{\phi} = g_s$$

DUAL TO

[BERENSTEIN MALDACENA NASTASE]

SECTOR OF $\mathcal{N}=4$ $d=4$ $SU(N)$ SUPER YANG MILLS WITH $N \to \infty$ AND $g^2_{YM} = 4\pi g_s$

THE SECTOR IS ONE OF LARGE "J" CHARGE UNDER A $U(1)$ SUBGROUP OF $SO(6)$ R SYMMETRY GROUP

DICTIONARY

$$\mu p^+ \alpha' = \frac{J}{\sqrt{\lambda}} = \frac{J}{\sqrt{g^2_{YM} N}} \ ; \ \frac{2p^-}{\mu} = \Delta - J.$$

NOTE : $J \sim \sqrt{N}$; $\Delta - J = O(1)$

FREE IIB THEORY ON "PP WAVE"
EASILY QUANTIZED (MATSAEV)

SPECTRUM

$$2p^- p^+ = \sum_{i=1}^{\infty} \sum_{n=0-\infty}^{\infty} N_n^i \sqrt{\frac{n^2}{\alpha'^2} + (\mu p^+)^2}$$

[SINGLE PARTICLE]

DUALITY PREDICTS OPERATORS
WITH

$$(\Delta - J) = \sum_{i=1}^{8} \sum_{n=-\infty}^{\infty} N_n^i \sqrt{1 + \frac{g_{YM}^2 N}{J^2} n^2}$$

[SINGLE TRACE]

BMN OPERATORS

E.G. $\sum_{k=0}^{J} \mathrm{Tr}\left[\psi Z^k \phi Z^{J-k}\right] e^{\frac{i n k}{J}}$

→ PRECISELY CHIRAL AT $n = 0$
APPROX CHIRAL AT $n \neq 0$

→ MANY "IMPURITIES" EACH WITH ITS OWN PHASE [$\sum n_i = 0$ ⟷ LEVEL MATCHING]

EVIDENCE FOR BMN IDENTIFICATION

DIRECT COMPUTATION OF $\Delta - J$ IN GAUGE THEORY [PLANAR GRAPHS]

: 1 LOOP ; AGREES WITH SPECTRUM (BMN)
: 2 LOOP ; ✓ [GROSS, MIKHAILOV, ROIBAN]
: ALL ORDERS (SOME ASSUMP) ; ✓ [SANTAMBROGIO, ZANON]

[NOTE : PT EXP IN $\lambda' = \frac{g^2_{YM} N}{J^2}$]

IMPORTANT ASSUMPTION

<u>PLANAR</u>

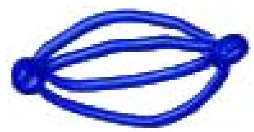
NAIVE : PLANAR APPROXIMATION EXACT BECAUSE $N \to \infty$ [INCORRECT]

: TO SEE WRONG NOTE PLANAR REPRODUCES FREE THEORY.
FULL STRING THEORY INTERACTING.

SO NONPLANAR GRAPHS MUST CONTRIBUTE EVEN IN THE STRICT $N \to \infty$ BMN LIMIT.
HOW?

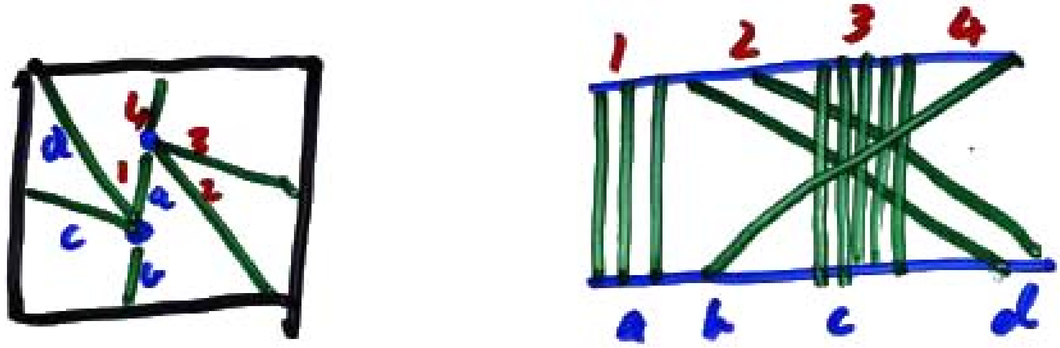
NUMBER OF GRAPHS AT GENUS $h \sim J^{4h} \underbrace{f(J)}_{\text{INDEP OF } h}$

GENUS 0 :



GRAPHS = J [CYCLICAL ROTNS]

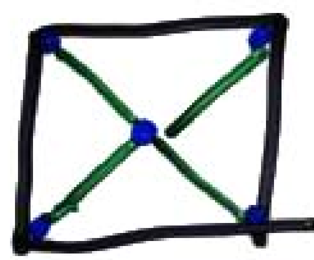
GENUS 1



$$\# \text{ GRAPHS} = \left({}^{J}C_4 + {}^{J}C_3\right) 8 \times J$$

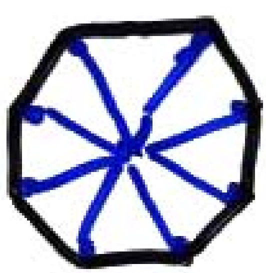
$$\sim \frac{J^4}{24} \times J \quad \text{(LEADING LARGE } J\text{)}$$

GENUS 1 REDRAWN



[47, KRISTJANSEN ET AL.]

GENUS II



SCALES LIKE J^{4h}

$$\langle \mathrm{Tr}\, Z^J(0)\, \mathrm{Tr}\, \bar{Z}^J(x) \rangle = \frac{1}{(4\pi^2 x^2)^J}\, \frac{2 \sinh(g_2/2)}{g_2} \qquad \left[g_2 = \frac{J^2}{N} \right]$$

SUMMARY

→ EACH GENUS h GRAPH $= O\left(\frac{1}{N^{2h}}\right)$

→ # OF SUCH GRAPHS $\sim J^{4h}$

[KRISTJANSEN ET AL, BERENSTEIN NASTASE, GT]

→ FULL CONTRIBUTION AT GENUS h

$\sim g_2^{2h}$; $g_2 = J^2/N$.

HIGHER GENUS GRAPHS ENCODE STRING INTERACTIONS

$$g_2^2 = g_s^2 (\mu p^+ \alpha')^4$$

$$= \frac{g_s^2 \alpha'^4}{L^8} \; ; \; L = \frac{1}{\sqrt{\mu p^+}} = \text{EFF COMPACTIFICATION}$$

= EFFECTIVE $d=2$ NEWTONS CONST.

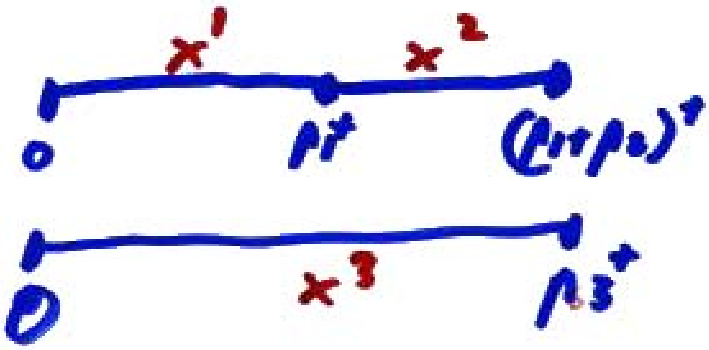
CONSEQUENTLY, GAUGE THEORY GENUS EXPANSION PARAMETER MAPS TO NATURAL STRINGY GENUS EXP PARAMETER.

REST OF TALK. MORE SYSTEMATIC EXP OF STRING INT.

3 STRING INTERACTIONS FROM 3 PT FTN

LIGHTCONE SFT WELL DEFINED AND COMPUTABLE IN PP-WAVE.

RECALL



[SPRADLIN VOLOVICH]

$$\langle \psi_3 | P^-_{INT} | \psi_1 \psi_2 \rangle =$$

$$\int Dx^1(\sigma)\, Dx^2(\sigma)\, Dx^3(\sigma)\; \psi_1[x^1(\sigma)]\, \psi_2[x^2(\sigma)]\, \psi_3^*[x^3(\sigma)]$$

$$\times \text{ PREFACTOR } \times \delta\left[x^1(\sigma)+x^2(\sigma)-x^3(\sigma)\right]$$

PROPOSAL [57]

$$\langle \psi_3 | P^-_{INT} | \psi_1 \psi_2 \rangle = \mu(\Delta_1+\Delta_2-\Delta_3)\, C_{123}$$

PROPOSAL VALID TO $O(\lambda')$ ONLY.

C_{123} = 3 PT COEFFICIENT IN FREE THEORY OF NORMALIZED BMN OPERATORS

ELABORATION

$\langle \psi | \phi \rangle \leftrightarrow \langle O_\psi \, O_\phi \rangle$ (STATE OPERATOR MAP)

REASONABLE THAT

$\langle \psi | P^- | \phi \rangle \leftrightarrow$ (LINEAR FACTOR IN P^-) $\langle O_\psi \, O_\phi \rangle$

LINEAR FACTOR MUST VANISH AT

$\lambda' = \dfrac{g^2_{YM} N}{J^2} = 0$ (ANOMALOUS DIM $= 0$)

FIXED TO BE $(\rho_1^- + \rho_2^- - \rho_3^-)$.

: SCALINGS : $C_{123} \sim \dfrac{J^{3/2}}{N} = \dfrac{J^2}{N} \times \dfrac{1}{\sqrt{J}}$

$$\langle \psi_3 | P^- | \psi_1 \psi_2 \rangle \sim \frac{g_2 \lambda' \mu}{\sqrt{J}}$$

NOTE : $\sum\limits_{\text{FIN STATES}} |\text{Amp}|^2 = |\text{Amp}|^2 \times J =$ FINITE

[BETTER : CHANGE NORMALIZATION]

: MATRIX ELEMENTS VANISH ON-SHELL.

$\Rightarrow$ EXCITED STRING STATES STABLE TO THIS ORDER.

COMPARISON WITH $\mu \to \infty$ SFT

RECALL: $$S = \frac{1}{4\pi}\int\left(\dot{x}^2 - x'^2 - (\mu p^+)^2 x^2\right) \quad (\alpha' = 1)$$

$$= \frac{1}{4\pi}\int\left(\dot{x}^2 - (\mu p^+)^2 x^2\right) \quad (\mu \to \infty)$$

SO AT $\mu = \infty$ STRING DISINTEGRATES INTO BITS THAT DONT TALK TO EACH OTHER. EACH BIT HAS $\omega = (\mu p^+)^2$.

3 STRING HAMILTONIAN IGNORING ~~INTERACTIONS~~ PREFACTOR

$$H_{INT} = \int \mathcal{D}x_1\, \mathcal{D}x_2\, \mathcal{D}x_3\; \psi_1[x_1]\, \psi_2[x_2]\, \psi_3[x_3]\; \delta[x_1 + x_2 - x_3]$$

$$= \prod_{i=1}^{J} \langle \phi_i | \psi_i \rangle$$

DISCRETIZING THE BIG STRING INTO J BITS. $|\psi_i\rangle$ IS THE STATE OF i^{th} BIT ON BIG STRING ($|\phi_i\rangle$ ON THE SMALL STRINGS)

$\equiv$ RULE FOR FREE PLANAR YANG MILLS CORRELATORS!

INCLUDING PREFACTOR

OUR CONJECTURE (G7)

PREFACTOR WILL MODIFY THIS SFT FORMULA INTO

$$\langle \psi_3 | P^- | \psi_1 \psi_2 \rangle = \mu \left(\Delta_1 + \Delta_2 - \Delta_3 \right) C_{123} .$$

$\left[O(\lambda') = O\left(\frac{1}{\mu^2}\right) \right]$

STRING FIELD THEORY IN THE PP-WAVE BACKGROUND ANALYSED IN DETAIL BY SPRADLIN - VOLOVICH.

DETAILED ANALYSIS EXPLICITLY VERIFIED THIS CONJECTURE.

INTERESTING OPEN QUESTION.

FIND THE CORRECT GENERALIZATION TO ALL ORDERS IN λ' (FINITE μ).

ONE LOOP MASS RENORMALIZATION FROM 2^{ND} ORDER PERTURBATION THEORY

$$a_\psi^{+n} a_\phi^{+-n} |J\rangle \longleftrightarrow \sum_{k=0}^{J} \mathrm{Tr}(\phi Z^k \psi Z^{J-k}) e^{\frac{ikn}{J}}$$

AT LOWEST ORDER IN λ' $\left(\frac{1}{\mu^2}\right)$ COMPUTE ITS MASS RENORMALIZATION:

INTERMEDIATE STATES THAT CONTRIBUTE (SEE NOTE AHEAD

- $a_\psi^{+m} a_\phi^{+-m} |J_1\rangle \otimes |J_2\rangle$
- $a_\psi^{+0} |J_1\rangle \otimes a_\phi^{+0} |J_2\rangle$

$$\Delta E_m = \sum_n \frac{|V_{mn}|^2}{E_m - E_n}$$

STRAIGHT FORWARD CALCULATION

$$\Delta E = \frac{-g_2{}^2 \lambda'}{4\pi^2} \left(\frac{1}{3} + \frac{5}{2\pi^2 n^2} \right)$$

MASS RENORMALIZATION FROM GAUGE THEORY

: OUR GENERAL PHILOSOPHY

~~GENUS~~ 1 LOOP (STRING THEORY) $\longleftrightarrow$ GENUS 1 GAUGE THEORY

: TO MATCH MASS RENORMALIZATION BEHIND G_7 COMPUTED 2PT FTN OF BMN OPERATORS TO ONE LOOP.

$$(4\pi^2 x^2)^{J+2} \langle \bar{O}_n^J(0)\, O_n^J(x)\rangle$$
$$= (1 + g_2^2 A_{nn})\left[1 - n^2\lambda' \ln(\Lambda^2 x^2)\right] + \frac{\lambda' g_2^2}{4\pi^2}\left(\frac{1}{3} + \frac{5}{2\pi^2 n^2}\right)\ln \Lambda^2 x^2$$

A_{nn} COMPUTED BUT UNIMPORTANT.

IF O_n^J WERE GOOD OPERATORS [DEFINITE SCALING DIMENSION] THEN THIS WOULD IMPLY

COMPARE WITH

$$\delta\Delta = \frac{\lambda' g_2^2}{4\pi^2}\left(\frac{1}{3} + \frac{5}{2\pi^2 n^2}\right) \qquad \delta p^- = \frac{\mu\lambda' g_2^2}{4\pi^2}\left(\frac{1}{3} + \frac{5}{2\pi^2 n^2}\right)$$

APPARENT SPECTACULAR AGREEMENT.

PUZZLE

HOW DO WE SEE THIS EXTRA PIECE IN THE QUANTUM MECHANICS

POSSIBILITIES

(a) IT APPEARS AS AN EXPLICIT H. VERLIN
"CONTACT TERM" IN THE 1-1 STRING HAMILTONIAN, AT $O(g_s^2)$

(b) THE 2^{ND} ORDER PERTURBATION COMPUTATIONS WE PERFORMED [redacted] SUBTLY RECEIVE CONTRIBUTIONS FROM OTHER INTERMEDIATE STATES WE HAVE IGNORED.

BOTH THESE POSSIBILITIES CAN BE TESTED IN SFT. WE ARE HARD AT WORK.

AN UNSOLVED PUZZLE

[redacted]

OUR RECENT (TENTATIVE) COMPUTATIONS INDICATE O_n^J IS **NOT** THE GOOD OPERATOR TO $O(g_2)$

WE FIND

$$\tilde{O}_n^J = O_n^J - g_2 \sum_{m,s} \frac{C(m,n,s)\left(\frac{m}{s}\right)}{\left(n + \frac{m}{s}\right)} :O_m^{J_1} O^{J-J_1}:$$

WHERE $:O_m^{J_1} O^{J-J_1}:$ = DOUBLE TRACE OPERATOR.

$$C(m,n,s) = \frac{s^{3/2}}{\sqrt{J}} \frac{\sqrt{1-s}}{\pi^2} \frac{\sin^2(\pi n s)}{(ns-m)^2} \begin{bmatrix} \text{FREE} \\ \text{3 PT} \\ \text{FTN} \end{bmatrix}$$

$$s = \frac{J_1}{J}$$

THE SCALING DIMENSION OF $\tilde{O}_n^J$

$$\delta\Delta = \frac{\lambda' g_2^2}{4\pi^2}\left(\frac{1}{3} + \frac{5}{2\pi^2 n^2}\right) + \frac{\lambda' g_2^2}{16\pi^2}\left(\frac{45}{8} + \frac{1}{16\pi^2 n^2}\right)$$

NEW EFFECT FROM MIXING

CONCLUSIONS

① STRING INTERACTION EFFECTS ~~AR~~ IN THE PP WAVE ARE ENCODED IN HIGHER GENUS GRAPHS IN YANG MILLS.
$\left[g_2 = \frac{J^4}{N^2} = \right.$ RENORMALIZED GENUS COUNTING PARAMETER]

② AT $\mu \to \infty$ 3 STRING INTERACTIONS ARE OBTAINED FROM YANG MILLS CORRELATORS FROM A SIMPLE FORMULA. $\left[\langle 3 | P^- | 1, 2 \rangle = (\Delta_1 + \Delta_2 - \Delta_3) C_{123} \right]$

③ THE EFFECTIVE STRING COUPLING IN THE PP BACKGROUND (ATLEAST FOR A CLASS OF PROCESSES) IS $g_2 \sqrt{\lambda'}$

④ MASS RENORMALIZATIONS (QM PERT THEORY) AGREE WITH PERTURBATIVE GENUS 1 ANOMALOUS DIMENSIONS ON IGNORING OPERATOR MIXING. THIS AGREEMENT IS SPOILT ON INCLUDING THIS EFFECT. [RESOLUTION ~~IS~~ IS A PUZZLE BUT A TRACTABLE ONE]