Tachyon Dynamics in Open String Theory

Goal:

Construction of time dependent solution in open string theory describing rolling of a generic tachyon away from its maximum.

1. METHOD OF CONSTRUCTION

2. SPECIFIC EXAMPLES

A. S. hep-th/0207105

P. Mukhopadhyay & A.S., to appear

Also A.S. hep-th/0203211, 0203265

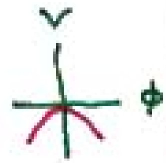
Moeller, Zwiebach: hep-th/0207109

→ similar analysis for p-adic string

Boundary SFT: Sugimoto, Terashima; Minahan

Consider an unstable D-brane system with a tachyon ϕ

$\rightarrow$ potential $V(\phi)$ has a maximum at $\phi = 0$.



For ordinary scalar field ϕ with standard 2-derivative action

$\rightarrow$ a two parameter family of classical solutions describing rolling of the tachyon away from the maximum.

$\rightarrow$ labelled by $\phi(x^0 = 0)$ and $\partial_0\phi(x^0 = 0)$.

Use time translation invariance to set either ϕ or $\partial_0\phi$ to 0 at $x^0 = 0$.

$\rightarrow$ a one parameter family of inequivalent solutions.

ENERGY

Q. Can we find a similar one parameter family of time dependent solutions describing rolling of an open string tachyon?

Our analysis will be at the level of classical open string theory ($g_s \to 0$)

General strategy:

1. Begin with a static solution that depends on some spatial coordinate x.

2. Replace x by $-ix^0$ where x^0 is the time coordinate.

3. The new configuration will be a solution of the equations of motion, but we need to make sure that the solution is real.

Suppose the tachyon has a mass$^2 = -m^2$.

Linearized equations of motion:

$$\boxed{(\partial_0^2 - m^2)\phi = 0}$$

1) $\phi = \lambda$, $\partial_0\phi = 0$ at $x^0 = 0$ gives

$$\boxed{\phi(x^0) = \lambda \cosh(m x^0)}$$

$E < V(0)$

2) $\phi = 0$, $\partial_0\phi = m\lambda$ at $x^0 = 0$ gives

$$\boxed{\phi(x^0) = \lambda \sinh(m x^0)}$$

$E > V(0)$

We shall concentrate on solutions of type 1) for definiteness, but the results generalize to type 2) initial conditions.

Wick rotation: $x^0 \to ix$

$$\boxed{\phi(x) = \lambda \cos(m x)}$$

GOAL: CONSTRUCT A ONE PARAMETER FAMILY OF SOLUTIONS LABELLED BY $\underline{\lambda}$ WHICH REDUCES TO $\underline{\phi = \lambda \cos(mx)}$ FOR <u>INFINITESIMAL λ</u>

$\leftrightarrow$ WILL BE DONE BY CONSTRUCTING A ONE PARAMETER FAMILY OF <u>BOUNDARY CFT</u> LABELLED BY $\underline{\lambda}$.

$V_\phi \equiv$ the vertex operator for zero momentum tachyon,

$\rightarrow$ switching on $\Phi(x) = \lambda \cos(mx)$ corresponds to a deformation of the CFT by

$$\boxed{\lambda \int_{boundary} dt \, \underbrace{\cos(mX(t)) V_\phi(t)}_{\text{dimension } 1}}$$

Special case:

$\cos(mX(t))V_\phi(t)$ is an exactly marginal operator

In this case the above perturbation describes a boundary CFT for finite λ.

$\rightarrow$ one parameter family of euclidean boundary CFT (BCFT)

$X \rightarrow -iX^0 \Rightarrow$ ROLLING TACHYON SOLUTION OF FULL STRING EQS. OF MOTION (NOT JUST OF LINEARIZED EQ.)

Generic case:

The operator $\cos(mX(t))V_\phi(t)$ is not exactly marginal (still has dimension 1)

$\rightarrow \beta_\lambda$ receives higher order corrections.

Consider now a different perturbation:

$$\boxed{\lambda \int dt \cos(\omega X(t))V_\phi(t)}$$

$$\boxed{\rightarrow \beta_\lambda = (\omega^2 - m^2)\lambda + g(\omega, \lambda)}$$

$g(\omega, \lambda)$: higher order contribution to β_λ

Now for every λ we adjust ω such that

$$\boxed{(\omega^2 - m^2)\lambda + g(\omega, \lambda) = 0}$$

$O(\lambda^3)$

$\rightarrow \omega = m + \mathcal{O}(\lambda^2)$

This describes a BCFT for every λ

$X \rightarrow -iX^0$: ROLLING TACHYON SOLN.

Note 1: For this procedure to work, OPE of $\cos(\omega X)V_\phi$ with itself should not generate any other operator of dimension $\simeq 1$ ($\sin(\omega x) V_\phi$, ∂x ruled out by $x \to -x$ sym)

Note 2: λ dependence of ω has an analogy in particle mechanics

Period of oscillation in general depends on the amplitude

Boundary state of this BCFT

→ energy momentum tensor (and other conserved currents associated with sources of other massless fields)
CLOSED STRING

Inverse Wick rotation ($x \to -ix^0$)

→ time evolution of energy momentum tensor associated with the rolling tachyon solution.

More explicit details for construction of the energy-momentum tensor:

$|\mathcal{B}\rangle$: boundary state associated with a boundary CFT → A CLOSED STRING STATE OF GHOST NO. 3

General form of contribution from level (1,1) states to $|\mathcal{B}\rangle$ (in bosonic string theory): in flat space

$$\boxed{\int d^{26}k[\tilde{A}_{\mu\nu}(k)\alpha^{\mu}_{-1}\bar{\alpha}^{\nu}_{-1} + \tilde{B}(k)(\bar{b}_{-1}c_{-1} + b_{-1}\bar{c}_{-1})] (c_0 + \bar{c}_0)c_1\bar{c}_1|k\rangle + \cdots\cdots}$$

$A_{\mu\nu}(x)$, $B(x)$

$\equiv$ Fourier transforms of $\tilde{A}_{\mu\nu}(k)$, $\tilde{B}(k)$

Then

$$\boxed{T_{\mu\nu}(x) = K(A_{\mu\nu}(x) + \eta_{\mu\nu}B(x))}$$

K: A normalization constant

$$Q_B|\mathcal{B}\rangle = 0 \Rightarrow \partial^{\mu}T_{\mu\nu} = 0$$

For a boundary CFT obtained by perturbation

$$\boxed{\lambda \int dt \cos(\omega X(t)) V_\phi(t)}$$

$T_{\mu\nu}$ takes the form:

$$\boxed{T_{\mu\nu}(x, \vec{x}) = \sum_{n=0}^{\infty} T_{\mu\nu}^{(n)}(\vec{x}) \cos(n\omega x)}$$

$\vec{x}$: 25 spatial coordinates of the original theory

x: Wick rotated time coordinate

Inverse Wick rotation: $x \to -ix^0$

$$\begin{aligned}
T_{00}(x^0, \vec{x}) &= -\sum_{n=0}^{\infty} T_{xx}^{(n)}(\vec{x}) \cosh(n\omega x^0), \\
T_{0i}(x^0, \vec{x}) &= i \sum_{n=0}^{\infty} T_{xi}^{(n)}(\vec{x}) \cosh(n\omega x^0), \\
T_{ij} &= \sum_{n=0}^{\infty} T_{ij}^{(n)}(\vec{x}) \cosh(n\omega x^0)
\end{aligned}$$

This gives the energy-momentum tensor associated with rolling tachyon solution.

This construction can be easily generalized to an unstable D-brane system in superstring theory.

Exact computation of $T^{(n)}_{\mu\nu}$ in the euclidean BCFT requires the BCFT to be solvable.

$\to$ typically true if $\cos(mX)V_\phi$ is exactly marginal

We shall now quote the results for several examples

(Valid to all orders in α', and λ but in $g_s \to 0$ limit)

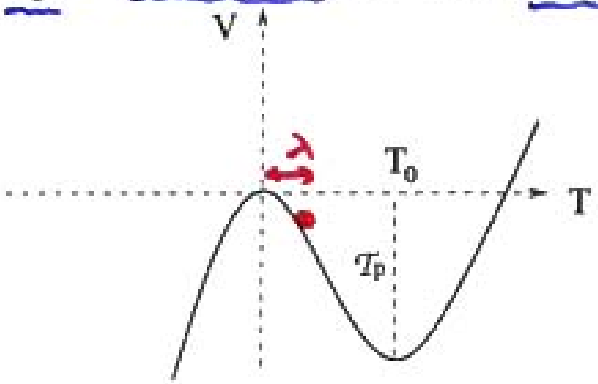
Example 1: Spatially homogeneous rolling tachyon on a single D-p-brane in bosonic string theory

Tachyon has mass$^2 = -1$ (in $\alpha' = 1$ unit)

Tachyon potential has a local minimum at $T = T_0$ on one side, but is unbounded from below on the other side.

$$\boxed{V(T_0) + \mathcal{T}_p = 0}$$

$\mathcal{T}_p$: Tension of the D-p-brane



Initial condition $T = \lambda$, $\partial_0 T = 0$ at $x^0 = 0$

$\rightarrow \quad T \simeq \lambda \cosh(x^0)$ to linear order ($E < \mathcal{T}_p$)

TOTAL ENERGY

Evolution of the energy momentum tensor:

$$\boxed{T_{00} = \frac{\mathcal{T}_p}{2}(1+\cos(2\pi\lambda))\ \delta(\vec{x}_\perp)}$$

$$\boxed{T_{i0} = 0, \qquad T_{ij} = -\mathcal{T}_p f(x^0)\delta_{ij}\ \delta(\vec{x}_\perp)}$$

$$\boxed{f(x^0) = \frac{1}{1+e^{x^0}\sin(\lambda\pi)} + \frac{1}{1+e^{-x^0}\sin(\lambda\pi)} - 1}$$

Note: for $\lambda > 0$, $f(x^0) \to 0$ as $x^0 \to \infty$

$\to$ Pressure (T_{ij}) vanishes

For $\lambda < 0$, $f(x^0)$ hits a singularity at

$$\boxed{x^0 = \ln \frac{1}{|\sin(\lambda\pi)|}}$$

This corresponds to pushing the tachyon to the wrong side where the potential is unbounded from below.

Euclidean BCFT analysis:

Callan, Klebanov, Ludwig, Maldacena
Polchinski, Thorlacius
Recknagel, Schomerus
Gaberdiel, Recknagel

Special case: $\lambda = \frac{1}{2}$

In this case

$$\boxed{T_{\mu\nu} = 0}$$

Interpretation: $\lambda = \frac{1}{2}$ corresponds to placing the tachyon at the minimum of the potential.

$\lambda = \frac{1}{2} + \kappa$ is equivalent to $\lambda = \frac{1}{2} - \kappa$.

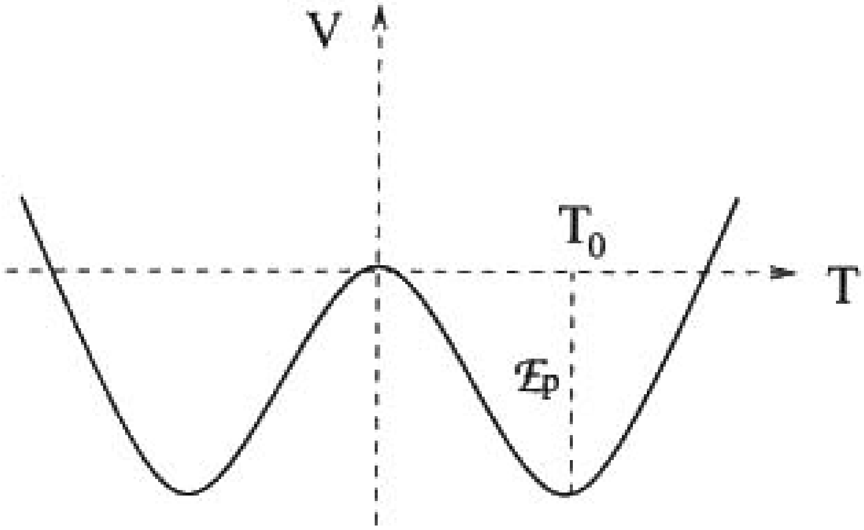
$\to$ we cannot place the tachyon beyond its vacuum value

(consistent with the boundary string field theory result that the vacuum is at $T = \infty$)

Example 2: Spatially homogeneous rolling tachyon on a non-BPS D-p-brane (or brane-antibrane system) in superstring theory

The tachyon has $\text{mass}^2 = -\frac{1}{2}$

Tachyon potential:



$\mathcal{E}_p$: Total energy density of the original system.

(Tension for non-BPS D-brane; 2×tension for brane-antibrane pair)

The tachyon begins rolling at $T = \lambda$, $\partial_0 T = 0$:

Thus to leading order $T = \lambda \cosh(x^0/\sqrt{2})$

Evolution of the energy-momentum tensor:

$$T_{00} = \frac{\mathcal{E}_p}{2}(1 + \cos(2\pi\lambda)) \; \delta(\vec{x}_\perp)$$

$$T_{i0} = 0, \qquad T_{ij} = -\mathcal{E}_p f(x^0)\delta_{ij} \; \delta(\vec{x}_\perp)$$

$$f(x^0) = \frac{1}{1 + e^{\sqrt{2}x^0}\sin^2(\lambda\pi)} + \frac{1}{1 + e^{-\sqrt{2}x^0}\sin^2(\lambda\pi)} - 1$$

NOTE

As $x^0 \to \infty$, $f(x^0) \to 0$ for either sign of λ

The system evolves to a gas of zero pressure.

Again $\lambda = \frac{1}{2}$ corresponds to placing the tachyon at the minimum of the potential.

Example 3: Rolling of a spatially homogeneous tachyon on a D-p-brane with electric field e along the x^1 direction.

P. Mukhopadhyay & A.S., to appear

(Consider bosonic string theory for definiteness, but the results easily generalize to non-BPS D-branes in superstring theory)

$\rightarrow$ can compute $T_{\mu\nu}$ and the source $S_{\mu\nu}$ for the antisymmetric tensor field $B_{\mu\nu}$

Result: When pushed in the right direction ($\lambda > 0$), $T_{\mu\nu}$ evolves to:

> $T_{\mu\nu}$ induced by fundamental strings + a gas of zero pressure and non-zero energy density.

Interpretation: Production of fundamental strings from the decay of a D-brane with electric field.

Effective field theory analysis:

Gibbons, Hori, Yi

also: Bergman, Hori, Yi

Harvey, Kraus, Larsen

Results for non-zero components of $\underline{T_{\mu\nu}}$, $\underline{S_{\mu\nu}}$:

($\delta(\vec{x}_\perp)$ omitted)

$$T_{00} = \left[\frac{1}{2} e^2 (1-e^2)^{-1/2} \mathcal{T}_p (1+\cos(2\pi\lambda))\right] + \left\{\frac{1}{2} (1-e^2)^{1/2} \mathcal{T}_p (1+\cos(2\pi\lambda))\right\},$$

$$T_{11} = -\left[\frac{1}{2} e^2 (1-e^2)^{-1/2} \mathcal{T}_p (1+\cos(2\pi\lambda))\right] - \left\{(1-e^2)^{1/2} \mathcal{T}_p f(\sqrt{1-e^2}\, x^0)\right\},$$

$$T_{ij} = -\left\{(1-e^2)^{1/2} \mathcal{T}_p f(\sqrt{1-e^2}\, x^0) \delta_{ij}\right\},\ 2 \le i, j \le$$

$$S_{01} = -S^{01} = -\left[e\,(1-e^2)^{-1/2} \mathcal{T}_p (1+\cos(2\pi\lambda))\right]$$

[]: Fundamental string, {}: rolling tachyon

$f(x^0)$: same function as before

$$f(x^0) = \frac{1}{1+e^{x^0}\sin(\lambda\pi)} + \frac{1}{1+e^{-x^0}\sin(\lambda\pi)} - 1$$

Example 4: Inhomogeneous rolling of the tachyon field

Consider a bosonic D-p-brane with one direction y compactified on a circle of radius $\sqrt{2}$.

First momentum mode T_1 of the tachyon:

$\rightarrow$ described by the vertex operator $V_\phi = \cos(Y/\sqrt{2})$ has dimension $1/2$

$\rightarrow$ describes a tachyonic mode of mass$^2 = -\frac{1}{2}$

We want to consider rolling of this tachyon away from its maximum. $(\phi = T_1)$

Solution of linearized equations:

$$\boxed{\phi(x^0) \simeq 2\lambda \cosh \frac{x^0}{\sqrt{2}}}$$

In terms of the original tachyon:

$$\boxed{T(x^0, y) \simeq 2\lambda \cosh \frac{x^0}{\sqrt{2}} \cos \frac{y}{\sqrt{2}}}$$

Answer for the energy-momentum tensor:

$$\boxed{\begin{aligned} T_{00} &= \frac{\mathcal{T}_p}{4}(\cos(2\lambda\pi)+1) \\ &\quad [f((x^0+iy)/\sqrt{2})+f((x^0-iy)/\sqrt{2})], \\ T_{0y} &= \frac{i\mathcal{T}_p}{4}(\cos(2\lambda\pi)+1) \\ &\quad [f((x^0-iy)/\sqrt{2})-f((x^0+iy)/\sqrt{2})] \\ T_{yy} &= -\frac{\mathcal{T}_p}{4}(\cos(2\lambda\pi)+1) \\ &\quad [f((x^0+iy)/\sqrt{2})+f((x^0-iy)/\sqrt{2})], \\ T_{ij} &= -\frac{\mathcal{T}_p}{4}\delta_{ij}\, f((x^0+iy)/\sqrt{2})\, f((x^0-iy)/\sqrt{2}) \end{aligned}}$$

where

$$\boxed{f(x^0) = \frac{1}{1+e^{x^0}\sin(\lambda\pi)} + \frac{1}{1+e^{-x^0}\sin(\lambda\pi)} - 1}$$

$\partial^\mu T_{\mu\nu} = 0 \to$ can be checked

T_{00} hits a delta-function singularity at

$$\boxed{\begin{aligned} x^0 &= \sqrt{2}\ln\left|\frac{1}{\sin(\lambda\pi/2)}\right| \equiv x^0_c, \\ y &= \begin{cases} \sqrt{2}\pi, & \text{for} \quad \lambda > 0, \\ 0, & \text{for} \quad \lambda < 0. \end{cases} \end{aligned}}$$

As x^0_c crosses this critical time, the system 'loses' a net amount of energy equal to its total energy at $y = 0$. or $\sqrt{2}\pi$.

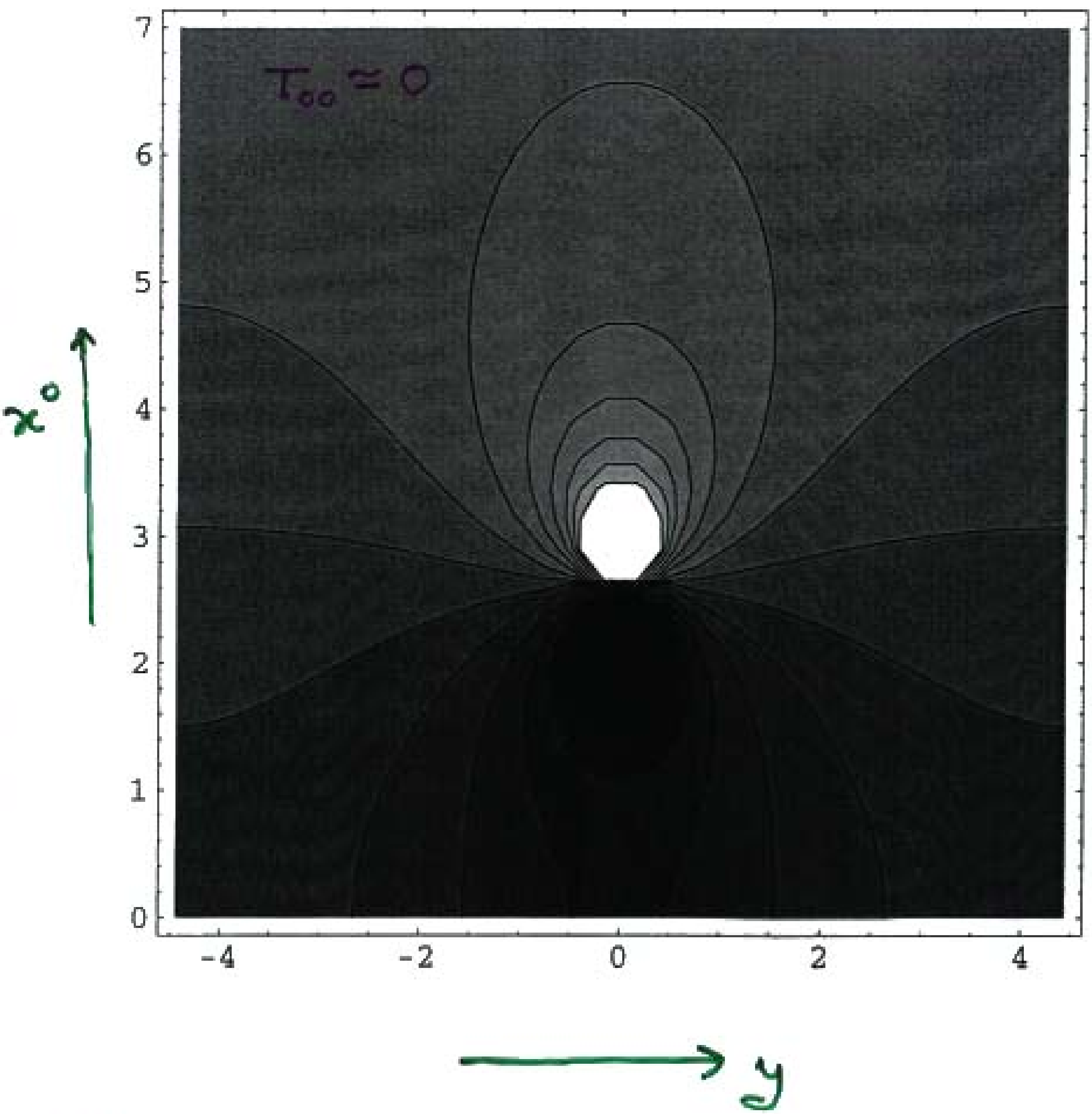
If we continue the evolution beyond x^0_c, eventually the energy density goes to zero everywhere.

Possible interpretation: Creation of a codimension 1 D-brane at $y = 0$ or $\sqrt{2}\pi$.

(No analogous results for superstring yet)

A contour plot of T_{00}: $\lambda < 0$

Darker region $\rightarrow$ Larger T_{00}



$\lambda > 0$: DIAGRAM SHIFTED BY $\sqrt{2}\pi$ ALONG y

GENERALIZATION: ROLLING OF MULTIPLE TACHYONS SIMULTANEOUSLY

VERTEX OP. $V_T^{(i)}$ OF DIMENSION $h_i < 1$

$$(\text{MASS})^2 = h_i - 1 \equiv -m_i^2 \qquad 1 \le i \le n$$

$\Rightarrow$ BOUNDARY PERTURBATION IN THE WICK ROTATED THEORY:

$$\int dt \left[\sum_i \lambda^{(i)} \cos\left(\omega^{(i)} x(t) + \varphi^{(i)}\right) V_T^{(i)}(t) \right]$$

2n PARAMETERS LABELLING 2n INITIAL CONDITIONS

NEED TO BE ADJUSTED TO MAKE β-FN = 0.

$$\Rightarrow \omega^{(i)} = m_i + O(\lambda^2)$$

CAUTION: OPE OF $\cos(\omega^{(i)} x + \varphi^{(i)}) V_T^{(i)}$ WITH EACH OTHER MUST NOT GENERATE OTHER OPERATORS OF DIMENSION $\simeq 2 + G(\lambda)$.

DANGEROUS OPERATORS:

$$\boxed{\sin(\omega^{(i)} x + \varphi^{(i)}) \, V_T^{(i)} \, , \quad \partial x}$$

THESE ARE NOT GENERATED IF $\omega^{(i)}$ ARE INCOMENSURATE

(SAME CONDITION THAT APPEARS IN PERTURBATION THEORY IN HAMILTONIAN SYSTEMS).

→ POINTS WHERE PERTURBATION THEORY FAILS ARE DENSE IN THE PARAMETER SPACE.

Summary

Wick rotation of euclidean BCFT's can be used to generate classical solutions in open string (field) theory, describing rolling of a tachyon away from its maximum.

If the euclidean BCFT is solvable then one can explicitly compute time evolution of the energy-momentum tensor and other conserved currents associated with the rolling tachyon solution.

If the euclidean BCFT is not solvable, one may still be able to use perturbative techniques to compute time evolution of $T_{\mu\nu}$ and other conserved currents.

→ NEEDS TO BE EXPLORED